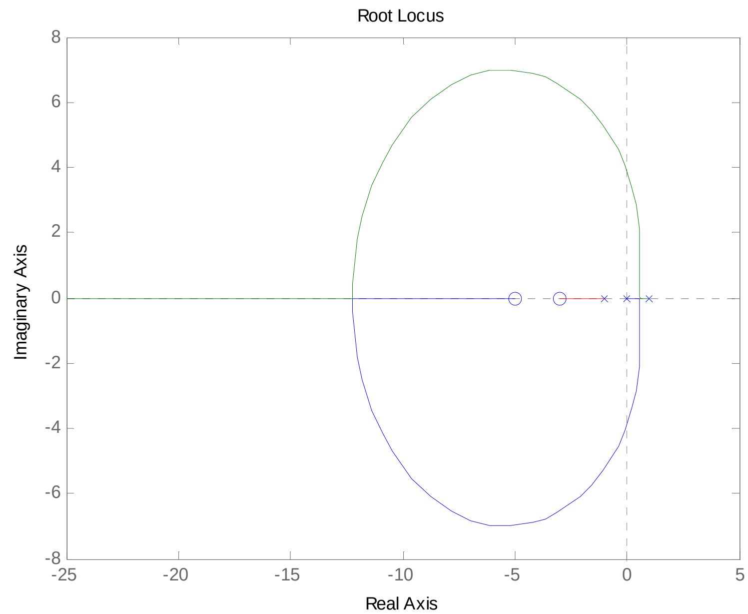
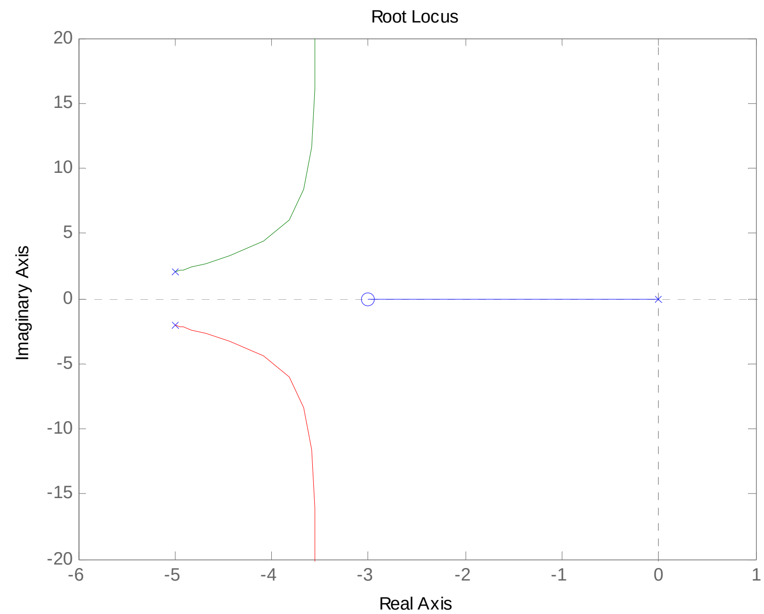
```
K=10;
%b)
num_G_b  = (3*K+K*s);
den_G_b =
(s^3+K*s^2+K*3*s-s);
G_b = num_G_b/den_G_b;
figure(2);
rlocus(G_b)
```



Root locus diagram, part (b):



7-30) Poles: $s = 0, -3.6$ zeros: $s = -0.4$

Angles of asymptotes: $\theta_i = \frac{2i+1}{3-1} \times 180 = 90^{\circ}, 270^{\circ}$

$$\sigma = -\frac{3.6-0.4}{3-1} = -1.6$$

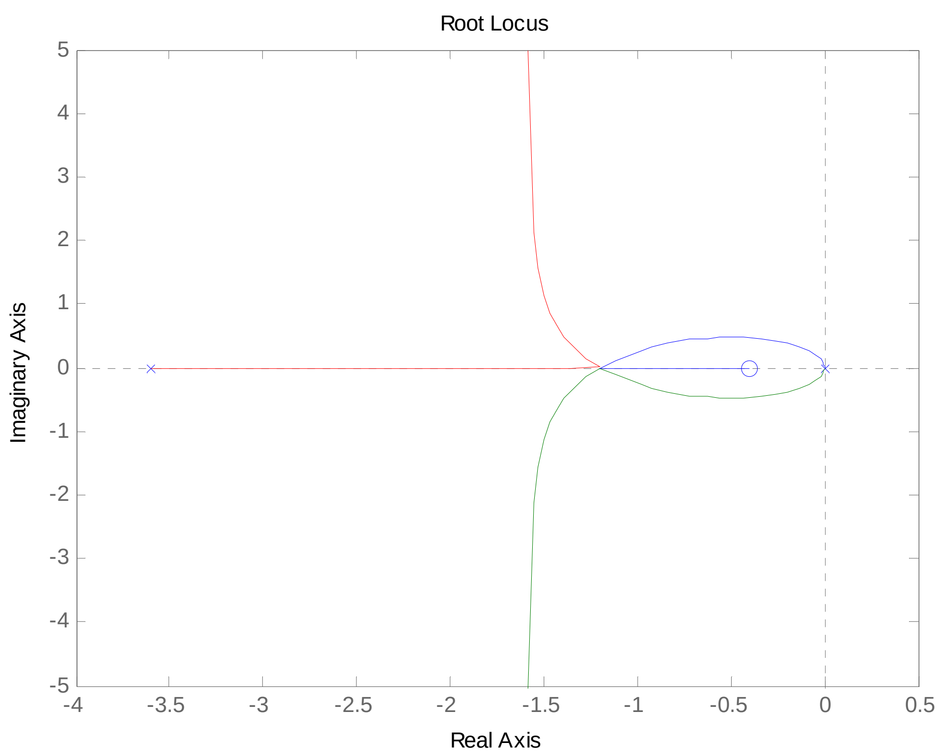
breakaway points: $\frac{1}{s^2} + \frac{1}{s+3.6} = \frac{1}{s+0.4}$

$\Rightarrow$ $s^3 + 2.4s^2 + 1.44s = 0$ $\rightarrow$ $s = 0, -1.2$

MATLAB code:

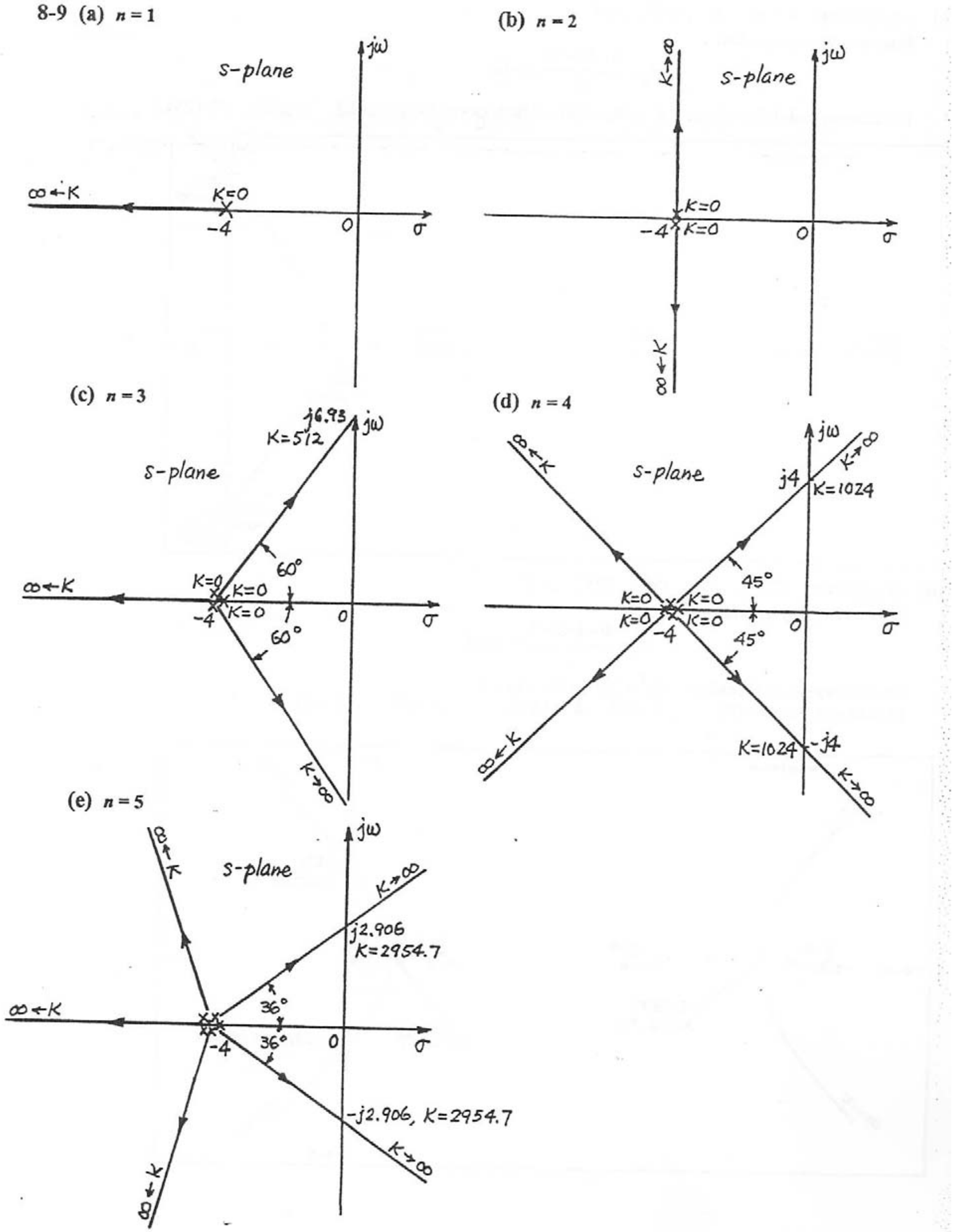
```
s = tf('s')
num_G=(s+0.4);
den_G=s^2*(s+3.6);
G=num_G/den_G;
figure(1);
rlocus(G)
```

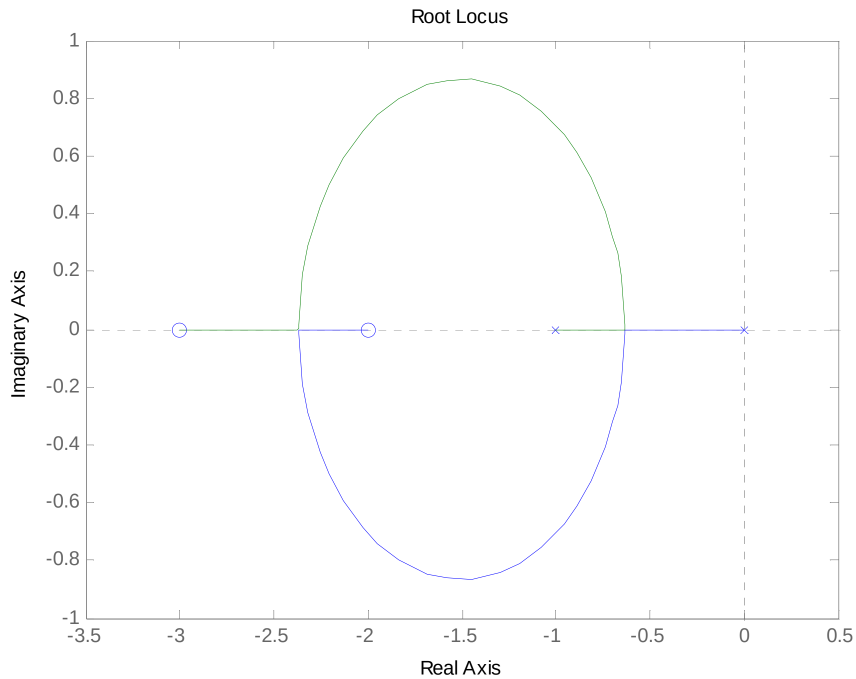
Root locus diagram:



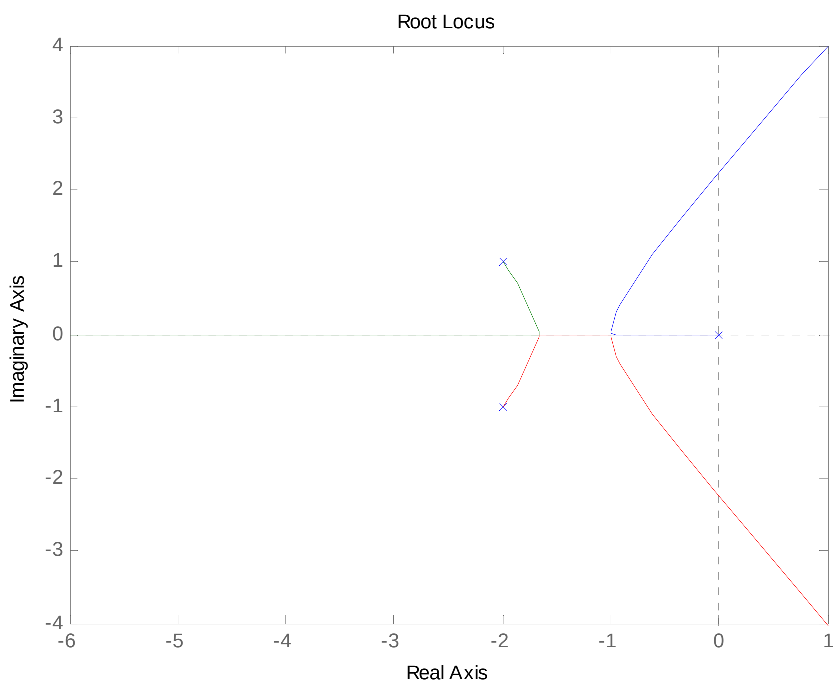
7-31 (a) $P(s) = s(s+12.5)(s+1)$ $Q(s) = 83.333$

7-20)


8-9 (a) n = 1
s-plane
jω
∞←K
K=0
-4
0
σ
(b) n = 2
K→∞
s-plane
jω
K=0
-4
K=0
0
σ
∞←K
(c) n = 3
j6.93
K=512
jω
s-plane
60°
∞←K
K=0
K=0
K=0
-4
0
σ
60°
K→∞
(d) n = 4
jω
∞←K
K→∞
s-plane
j4
K=1024
45°
K=0
K=0
K=0
K=0
-4
0
σ
45°
K=1024
-j4
∞←K
K→∞
(e) n = 5
jω
∞←K
s-plane
K→∞
j2.906
K=2954.7
36°
∞←K
36°
-4
0
σ
-j2.906, K=2954.7
K→∞
∞←K



Root Locus diagram – 7-17(i): (K = 2.93 @ damping = ~0.0707)

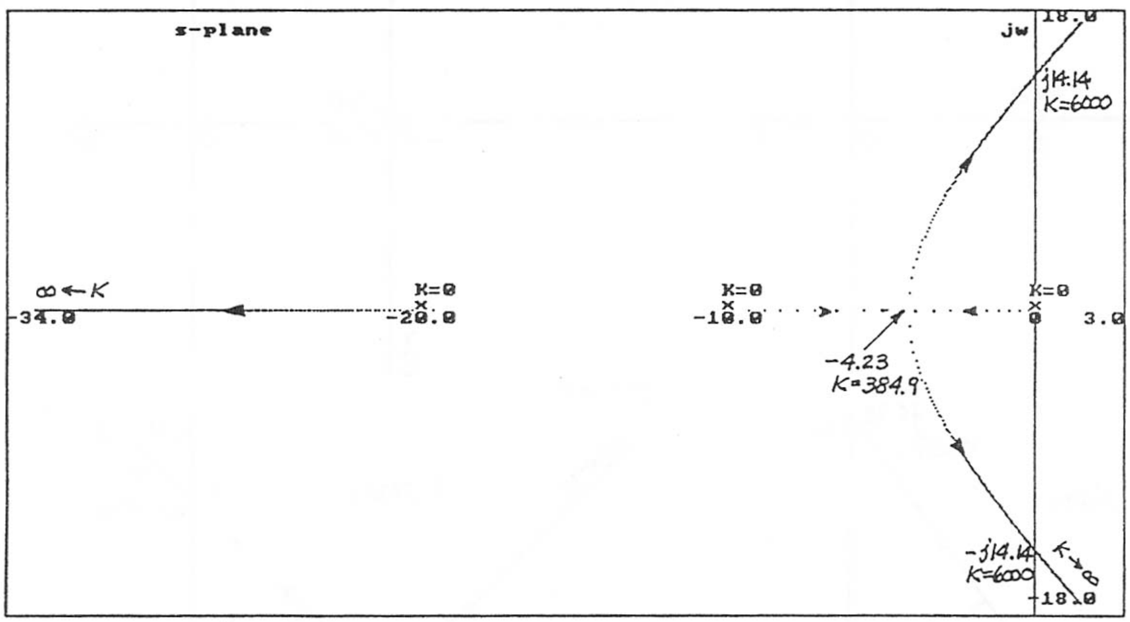


7-18) (a) Asymptotes: ***K* > 0:** 60^o, 180^o, 300^o

Intersect of Asymptotes:

$$\sigma_1 = \frac{0-10-20}{3} = -10$$

Breakaway-point Equation: $3s^2 + 60s + 200 = 0$ **Breakaway Point: (RL)** –4.2265, ***K* = 384.9**



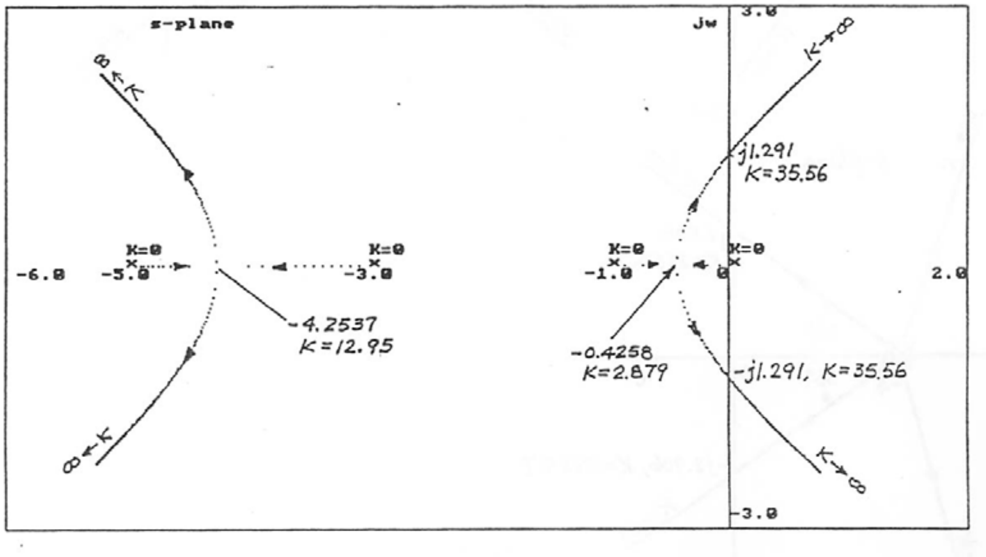
(b) Asymptotes: ***K* > 0:** 45^o, 135^o, 225^o, 315^o

Intersect of Asymptotes:

$$\sigma_1 = \frac{0-1-3-5}{4} = -2.25$$

Breakaway-point Equation: $4s^3 + 27s^2 + 46s + 15 = 0$

Breakaway Points: (RL) –0.4258 ***K* = 2.879**, –4.2537 ***K* = 12.95**



c) Zeros: $s = 0.5$ and poles: $s = 1$

Angle of asymptotes: $\theta = (2l + 1)180 = 180$

The breakaway points: $\dfrac{1}{(s+1)^2} = \dfrac{1}{s+0.5} \rightarrow s^2 + s + 0.5 = 0$

Then $s = -0.5 - 0.5j, -0.5 + 0.5j$ and $\sigma_1 = \dfrac{+1 - 0.5}{1} = 0.5$

d) Poles: $s = -0.5, 4.5$

Angle of asymptotes: $\theta_i = \dfrac{2l + 1}{2} \times 180 = 90, 270$

breakaway points:

$s^2 + s + 0.75 = 0 \rightarrow s = -1 - \sqrt{2}j, -1 + \sqrt{2}j$

$\sigma_1 = \dfrac{-0.5 + 1.5}{2} = 0.5$

e) Zeros: $s = -\dfrac{1}{3}, -1$ and poles: $s = 0, 0.5, 1$

Angle of asymptotes: $\theta_i = \dfrac{2l + 1}{3 - 2}180 = 180$

breakaway points: $\frac{1}{s}+\frac{1}{s+\frac{1}{2}}+\frac{1}{s-1}=\frac{1}{s+\frac{1}{3}}+\frac{1}{s+1} \rightarrow s=0.383,-2.22$

$$\sigma=-\frac{1-0.5+\frac{1}{3}+1}{1}=-\frac{11}{6}$$

f) Poles: $s=0,-3+4j,-3-4j$

Angles of asymptotes: $\theta_i=\frac{2i+1}{3}\times 180=60,180,300$

$$\sigma_1=-\frac{0+3-4j+3+4j}{3}=2$$

breakaway point: $-\frac{d}{ds}[s(s^2+6s+25)]=0$

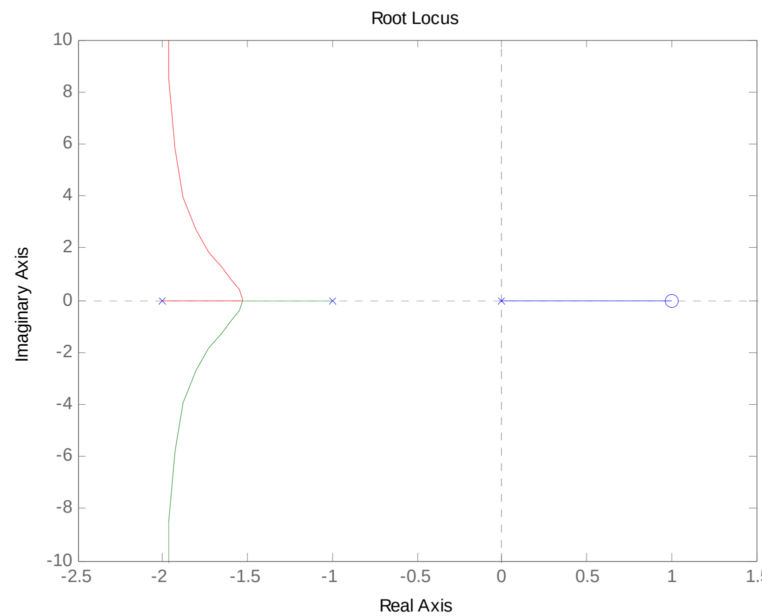
$3s^2+12s+25=0 \rightarrow s\approx -2+2.1j,-2-2.1j$

7-19) MATLAB code:

```
clear all;
close all;
s = tf('s')

%a)
num_G_a=1;
den_G_a=s*(s+10)*(s+20);
G_a=num_G_a/den_G_a;
figure(1);
rlocus(G_a)

%b)
num_G_b= 1;
den_G_b=s*(s+1)*(s+3)*(s+5);
G_b=num_G_b/den_G_b;
figure(2);
rlocus(G_b)

%c)
num_G_c=(s-0.5);
den_G_c=(s-1)^2;
G_c=num_G_c/den_G_c;
figure(3);
rlocus(G_c)

%d)
```

7-13(o) MATLAB code:

```
num=conv([1 5],[1 40]);
den=conv([1 0],[1 0]);
den=conv(den,[1 0]);
den=conv(den,[1 100]);
den=conv(den,[1 200]);
mysys=tf(num,den)
rlocus(mysys);
```



7-14) (a) $Q(s) = s + 5 \qquad P(s) = s\left(s^2 + 3s + 2\right) = s(s+1)(s+2)$

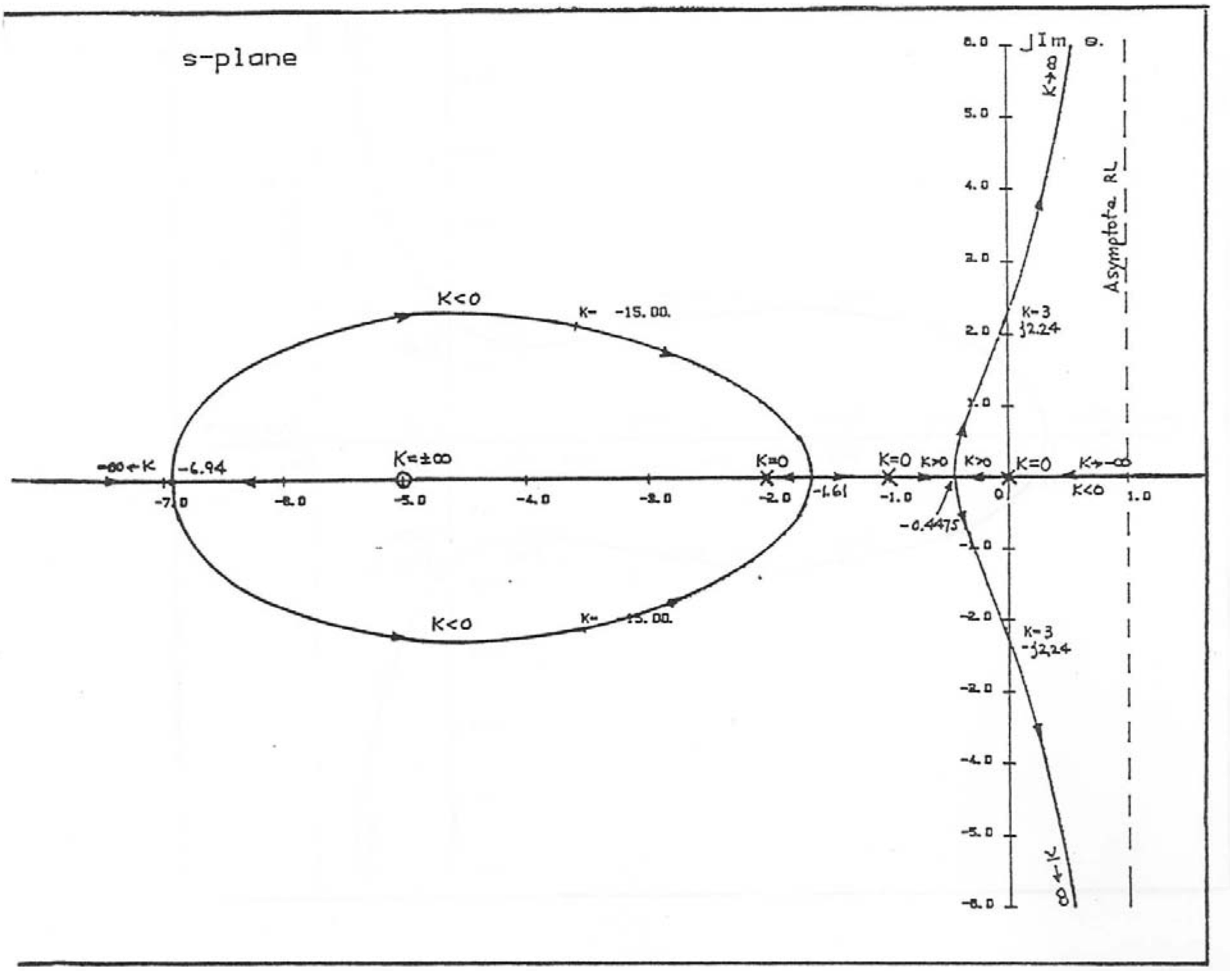
Asymptotes: ***K* > 0:** $90^o, \quad 270^o$ ***K* < 0:** $0^o, \quad 180^o$

Intersect of Asymptotes:

$$\sigma_1 = \frac{-1-2-(-5)}{3-1} = 1$$

Breakaway-point Equation: $s^3 + 9s^2 + 15s + 5 = 0$

Breakaway Points: $-0.4475, \quad -1.609, \quad -6.9434$



7-14 (b) $Q(s) = s + 3$ $P(s) = s(s^2 + s + 2)$

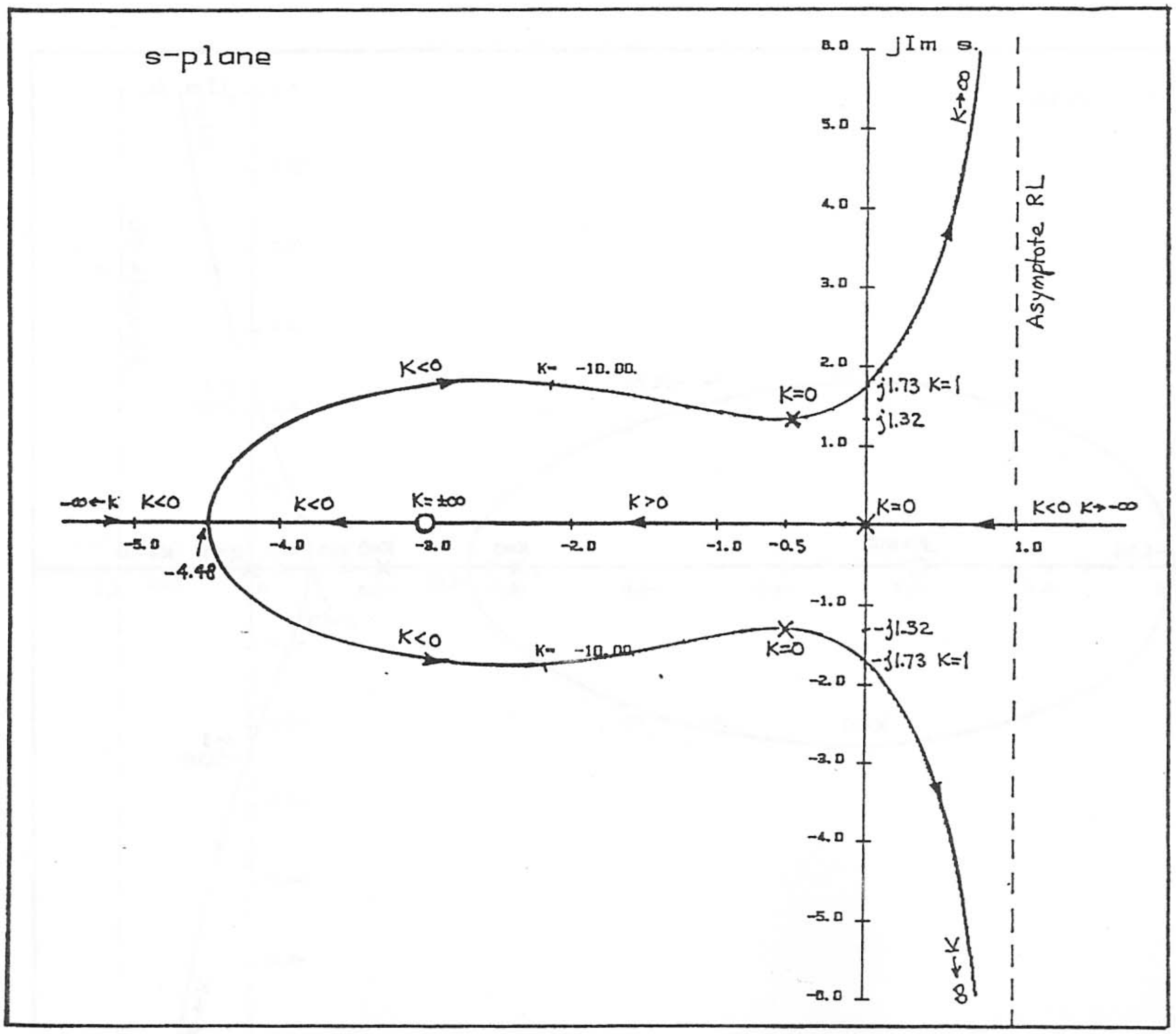
Asymptotes: ***K* > 0:** $90^o, \quad 270^o$ ***K* < 0:** $0^o, \quad 180^o$

Intersect of Asymptotes:

$$\sigma_1 = \frac{-1-(-3)}{3-1} = 1$$

Breakaway-point Equation: $s^3 + 5s^2 + 3s + 3 = 0$

Breakaway Points: –4.4798 The other solutions are not breakaway points.

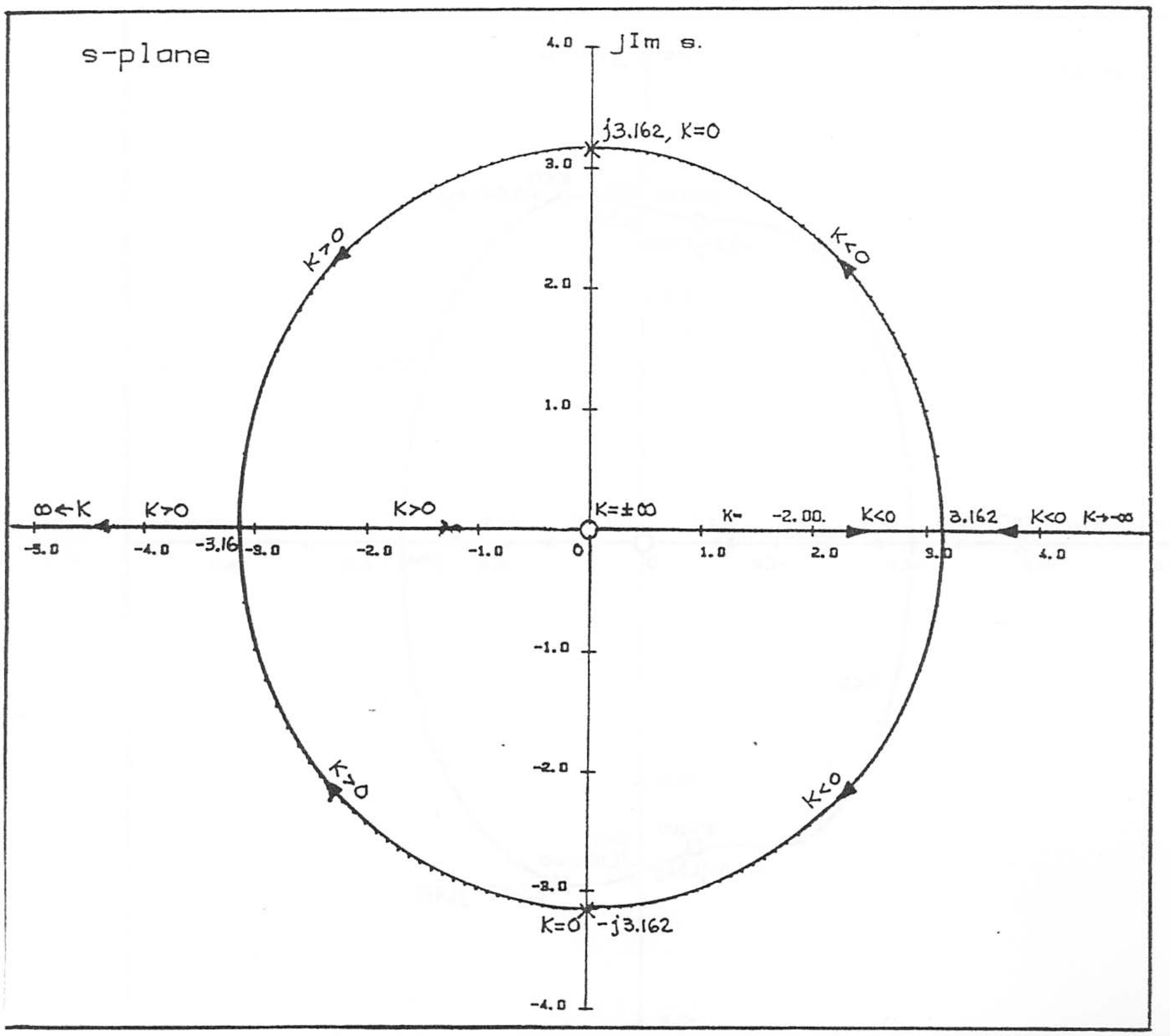


7-14 (c) $Q(s) = 5s \qquad P(s) = s^2 + 10$

Asymptotes: ***K* > 0:** 180^o ***K* < 0:** 0^o

Breakaway-point Equation: $5s^2 - 50 = 0$

Breakaway Points: –3.162, 3.162

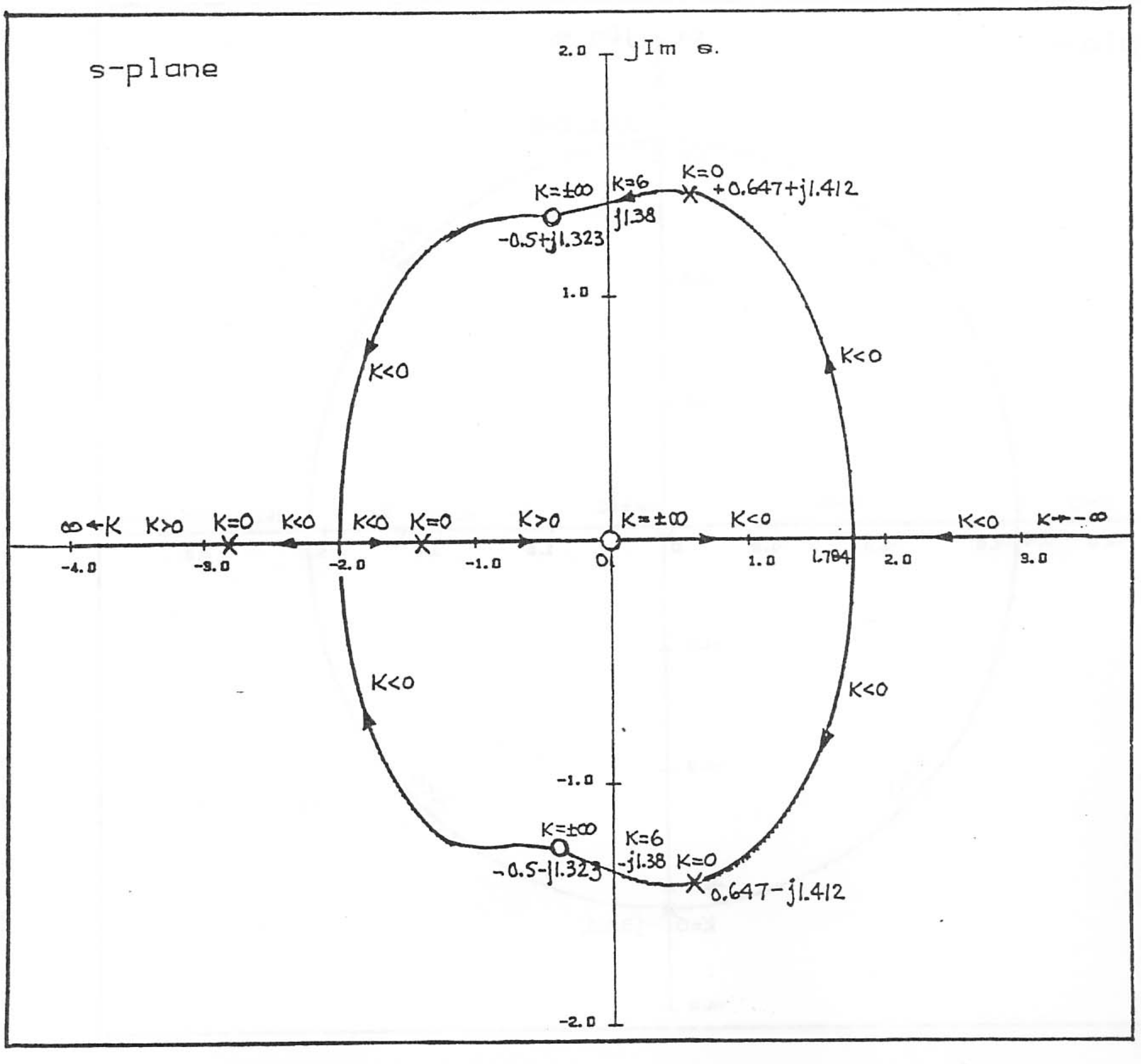


7-14 (d) $Q(s) = s\left(s^2 + s + 2\right)$ $P(s) = s^4 + 3s^3 + s^2 + 5s + 10$

Asymptotes: ***K* > 0:** 180^o ***K* < 0:** 0^o

Breakaway-point Equation: $s^6 + 2s^5 + 8s^4 + 2s^3 - 33s^2 - 20s - 20 = 0$

Breakaway Points: –2, 1.784. The other solutions are not breakaway points.

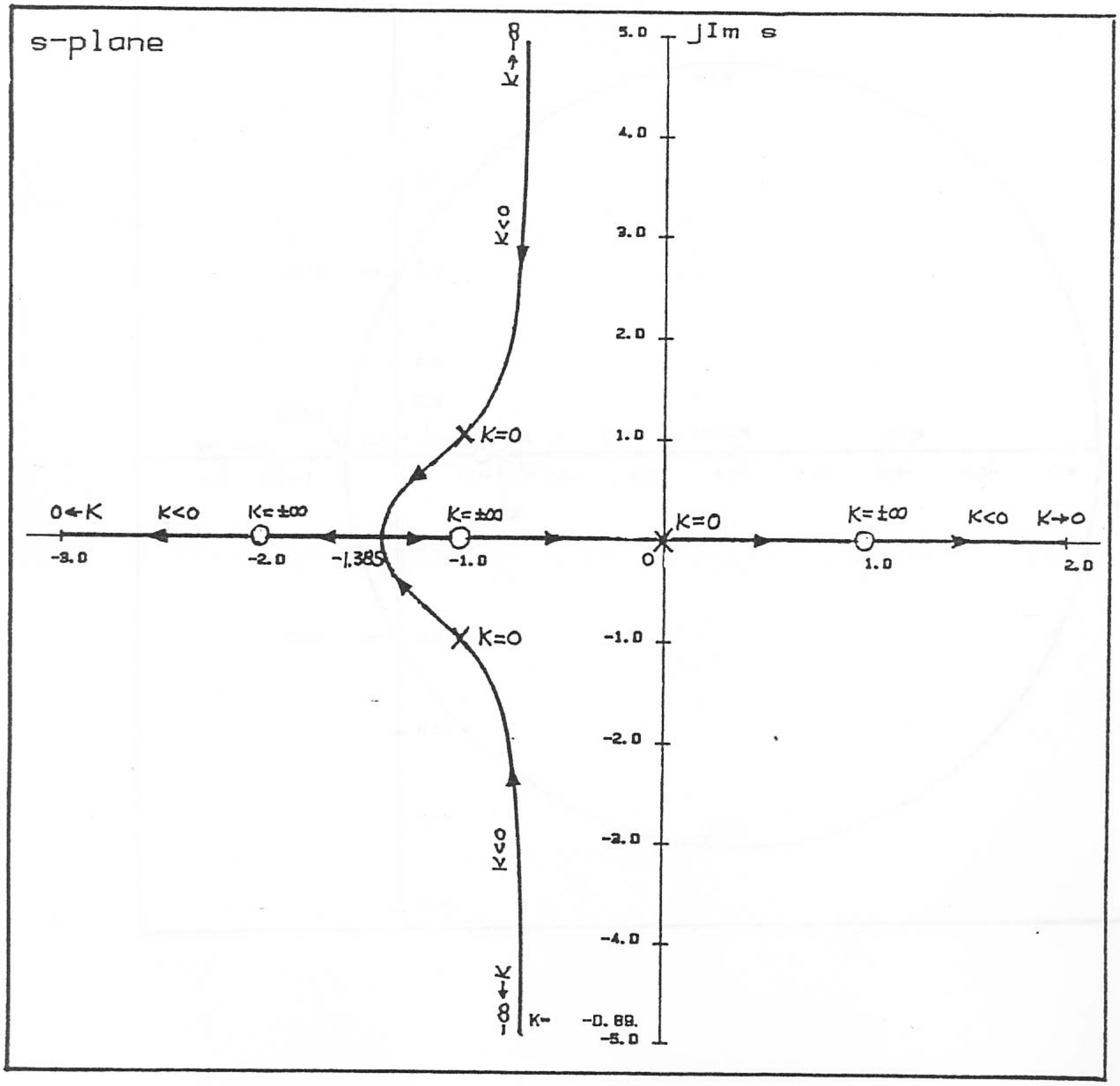


7-14 (e) $Q(s) = \left(s^2 - 1\right)\left(s + 2\right) \qquad P(s) = s\left(s^2 + 2s + 2\right)$

Since $Q(s)$ and $P(s)$ are of the same order, there are no asymptotes.

Breakaway-point Equation: $6s^3 + 12s^2 + 8s + 4 = 0$

Breakaway Points: –1.3848

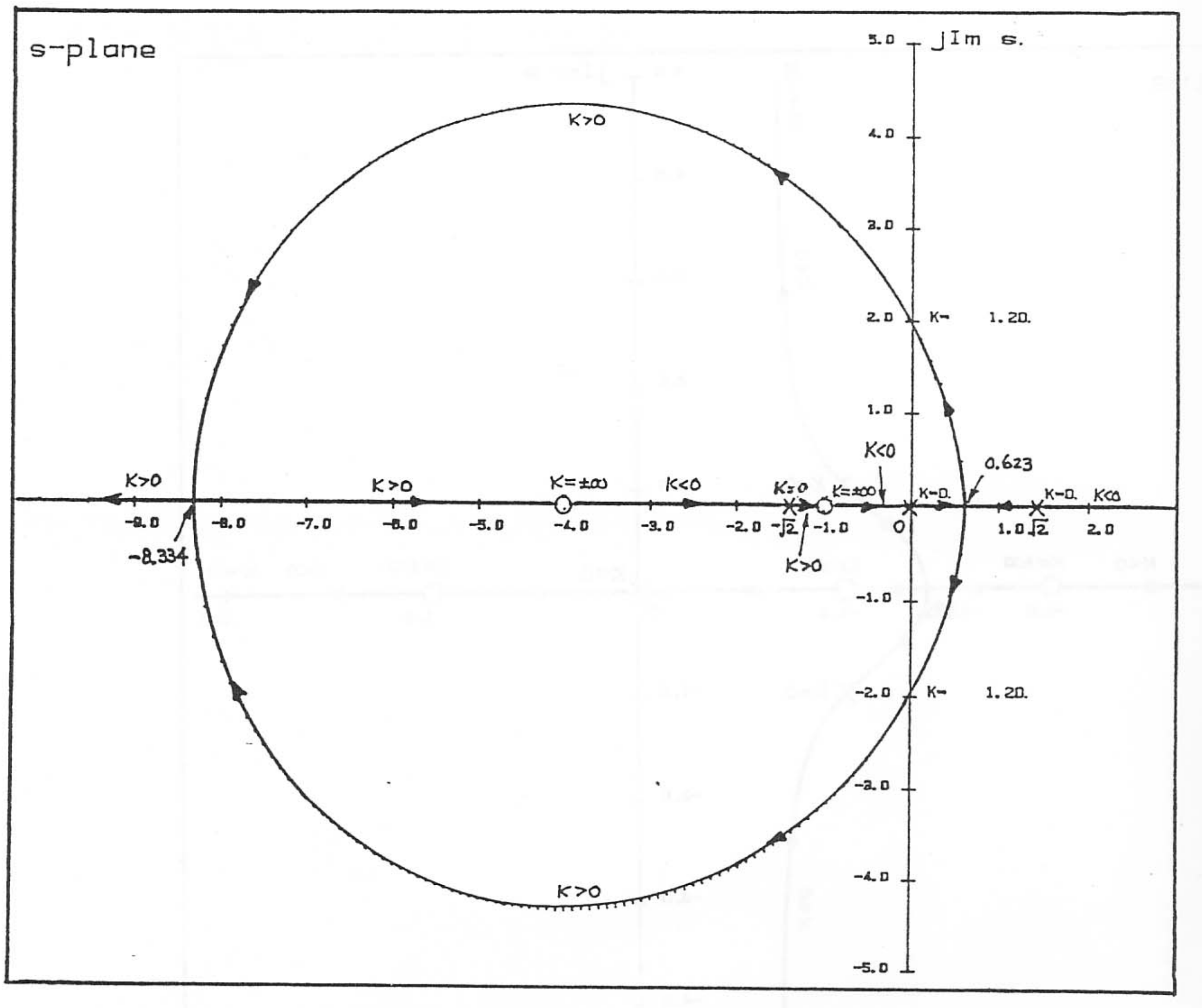


7-14 (f) $Q(s) = (s+1)(s+4)$ $P(s) = s\left(s^2 - 2\right)$

Asymptotes: ***K* > 0:** 180^o ***K* < 0:** 0^o

Breakaway-point equations: $s^4 + 10s^3 + 14s^2 - 8 = 0$

Breakaway Points: –8.334, 0.623



7-14 (g) $Q(s) = s^2 + 4s + 5 \qquad P(s) = s^2\left(s^2 + 8s + 16\right)$

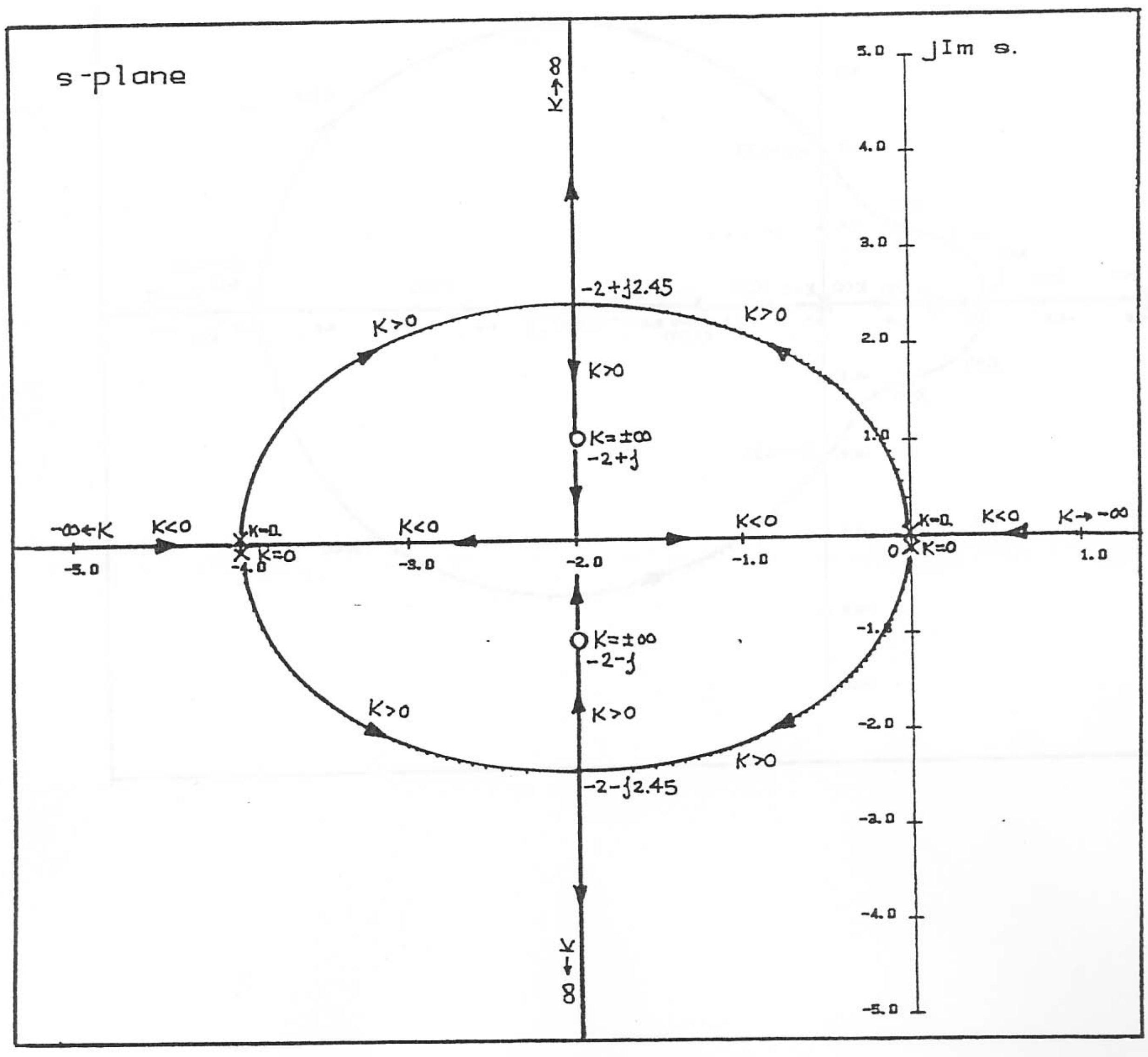
Asymptotes: ***K* > 0:** $90^o, \quad 270^o$ ***K* < 0:** $0^o, \quad 180^o$

Intersect of Asymptotes:

$$\sigma_1 = \frac{-8-(-4)}{4-2} = -2$$

Breakaway-point Equation: $s^5 + 10s^4 + 42s^3 + 92s^2 + 80s = 0$

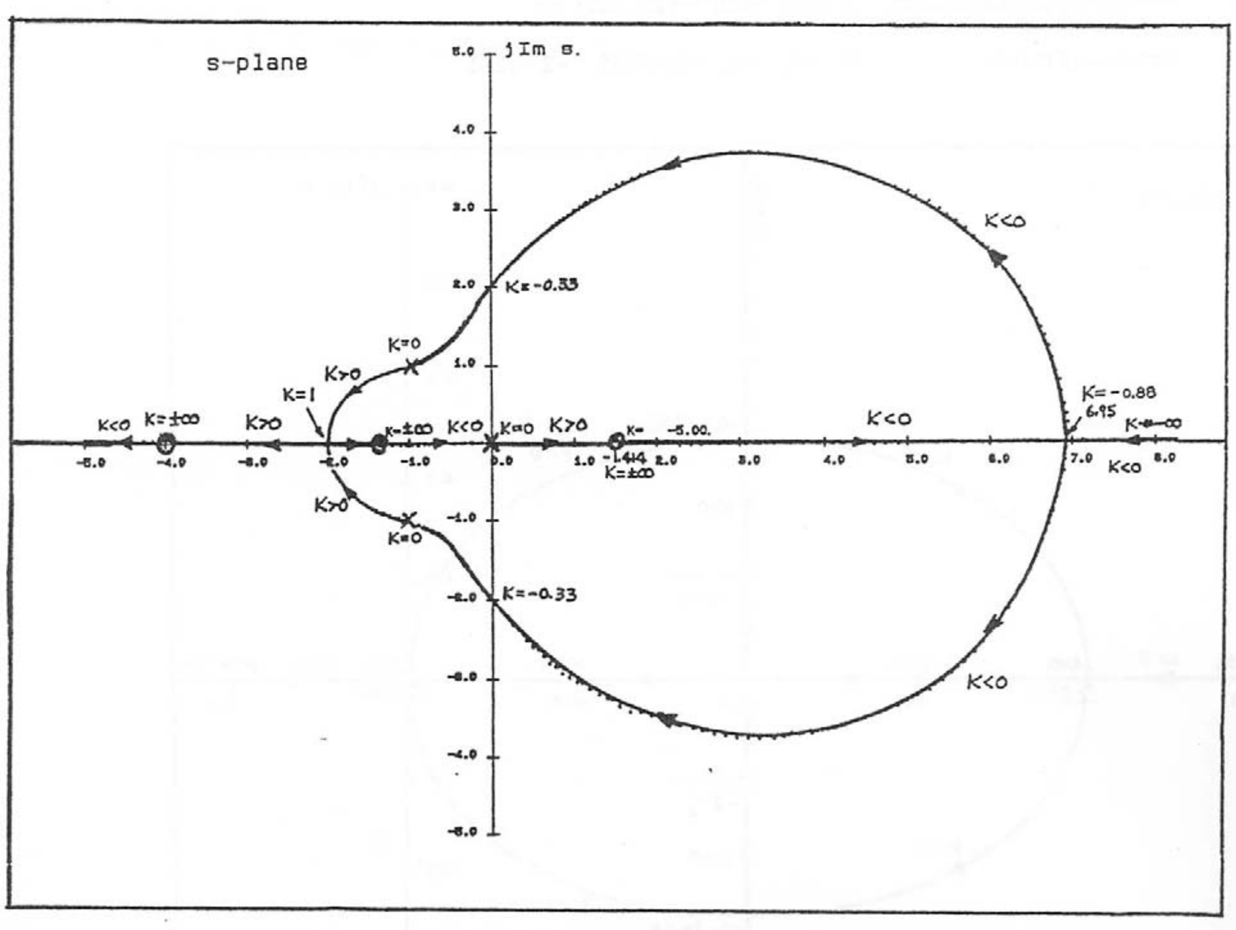
Breakaway Points: $0, \quad -2, \quad -4, \quad -2 + j2.45, \quad -2 - j2.45$



7-14 (h) $Q(s) = \left(s^2 - 2\right)\left(s + 4\right)$ $P(s) = s\left(s^2 + 2s + 2\right)$

Since $Q(s)$ and $P(s)$ are of the same order, there are no asymptotes.

Breakaway Points: –2, 6.95

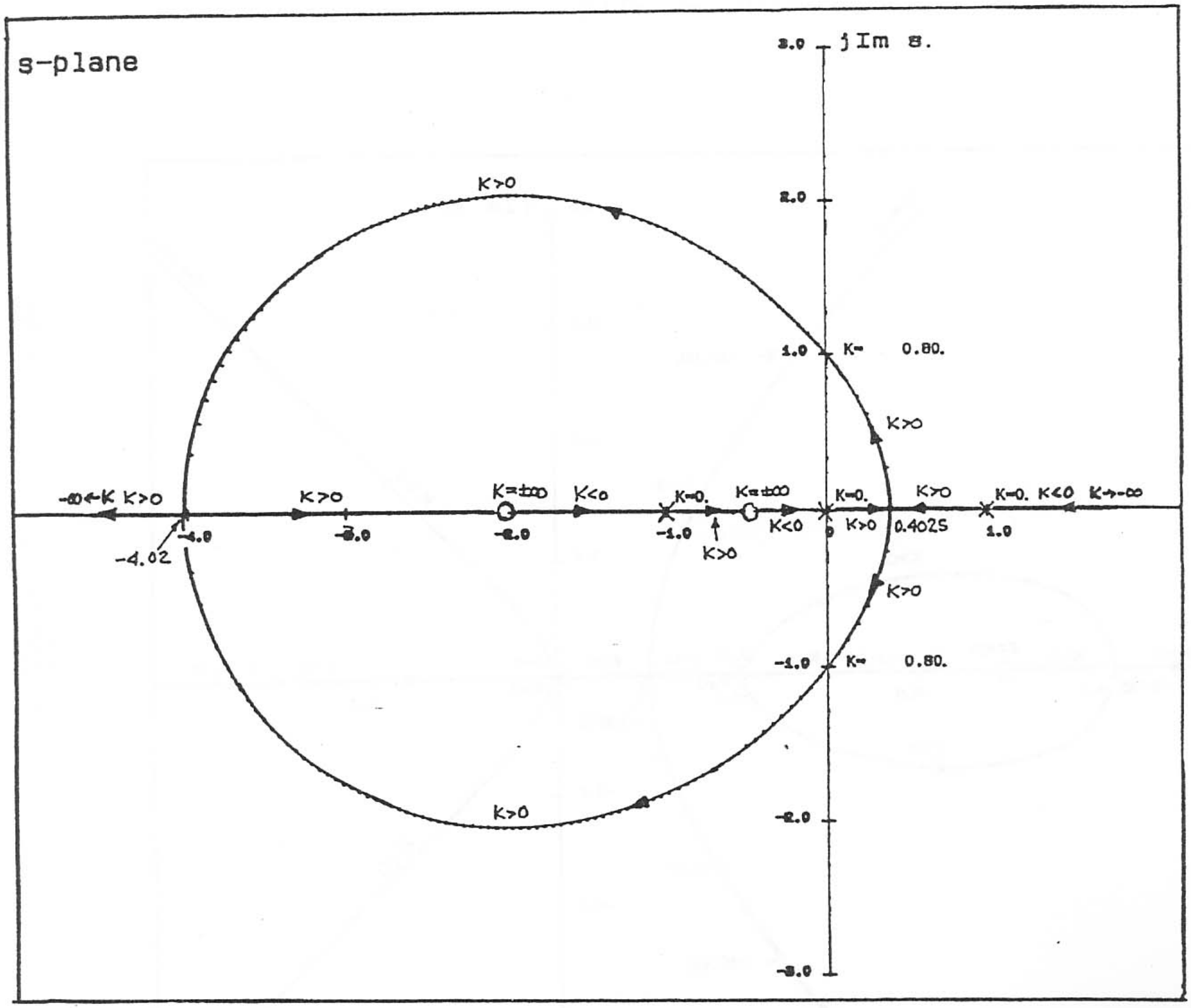


7-14 (i) $Q(s) = (s+2)(s+0.5) \qquad P(s) = s(s^2 - 1)$

Asymptotes: ***K* > 0:** 180^o ***K* < 0:** 0^o

Breakaway-point Equation: $s^4 + 5s^3 + 4s^2 - 1 = 0$

Breakaway Points: –4.0205, 0.40245 The other solutions are not breakaway points.


s-plane
jIm s.
K>0
K= 0.80.
K>0
-∞←K K>0
K>0
K=±∞
K<0
K=0.
K=±∞
K=0.
K>0
K=0. K<0
K→-∞
K<0
K>0
0.4025
K>0
-4.02
K>0
K= 0.80.
K>0

7-14 (j) $Q(s) = 2s + 5 \qquad P(s) = s^2\left(s^2 + 2s + 1\right) = s^2\left(s+1\right)^2$

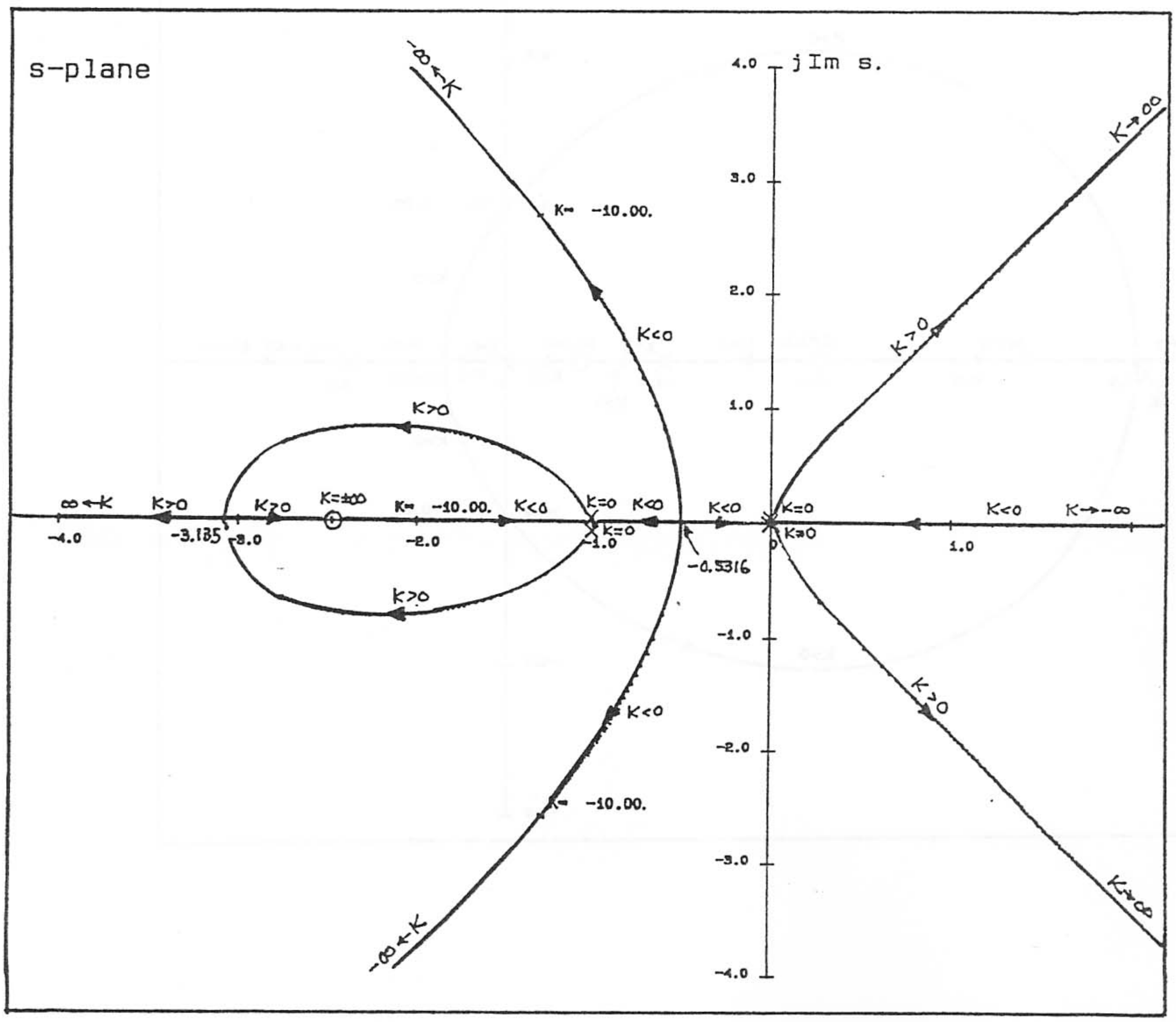
Asymptotes: ***K* > 0:** $60^o, \quad 180^o, \quad 300^o$ ***K* < 0:** $0^o, \quad 120^o, \quad 240^o$

Intersect of Asymptotes;

$$\sigma_1 = \frac{0+0-1-1-(-2.5)}{4-1} = \frac{0.5}{3} = 0.167$$

Breakaway-point Equation: $6s^4 + 28s^3 + 32s^2 + 10s = 0$

Breakaway Points: 0, –0.5316, –1, –3.135



7-15) MATLAB code:

```
clear all;
close all;
s = tf('s')

%a)
num_GH_a=(s+5);
den_GH_a=(s^3+3*s^2+2*s);
GH_a=num_GH_a/den_GH_a;
figure(1);
rlocus(GH_a)
```

Breakaway Points: $-2.5, \quad 1.09$

(d) Breakaway-point Equation: $-s^6 - 8s^5 - 19s^4 + 8s^3 + 94s^2 + 120s + 48 = 0$

Breakaway Points: $-0.6428, \quad 2.1208$

7-12) (a)

$$G(s)H(s) = \frac{K(s+8)}{s(s+5)(s+6)}$$

Asymptotes: *K* > 0: 90^o and 270^o ***K* < 0:** 0^o and 180^o

Intersect of Asymptotes:

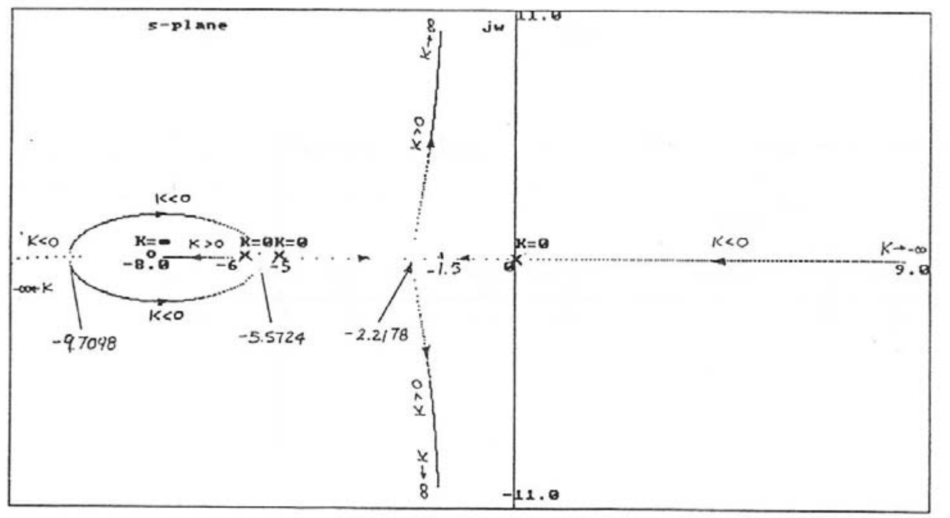
$$\sigma_1 = \frac{0-5-6-(-8)}{3-1} = -1.5$$

Breakaway-point Equation:

$$2s^3 + 35s^2 + 176s + 240 = 0$$

Breakaway Points: $-2.2178, \quad -5.5724, \quad -9.7098$

Root Locus Diagram:



7-12 (b)

$$G(s)H(s) = \frac{K}{s(s+1)(s+3)(s+4)}$$

Asymptotes: *K* > 0: $45^o, \quad 135^o, \quad 225^o, \quad 315^o$ ***K* < 0:** $0^o, \quad 90^o, \quad 180^o, \quad 270^o$

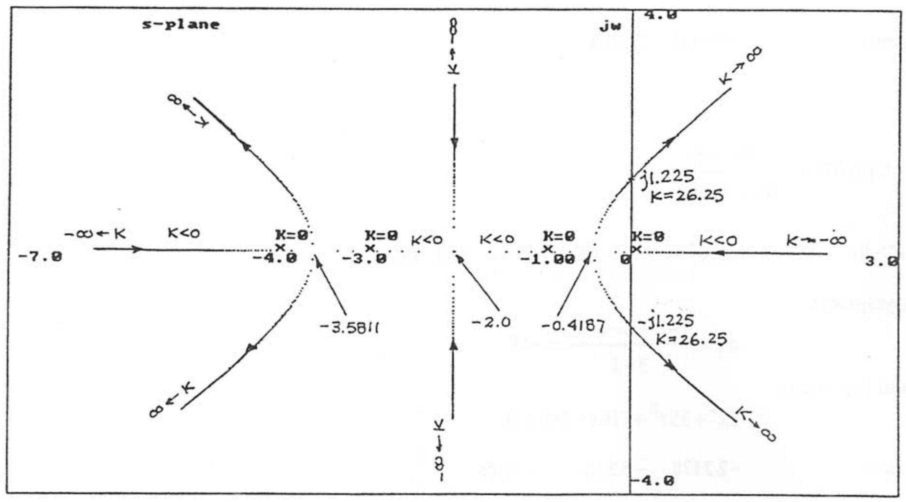
Intersect of Asymptotes:

$$\sigma_1 = \frac{0-1-3-4}{4} = -2$$

Breakaway-point Equation: $4s^3 + 24s^2 + 38s + 12 = 0$

Breakaway Points: $-0.4189, \quad -2, \quad -3.5811$

Root Locus Diagram:



7-12 (c)

$$G(s)H(s) = \frac{K(s+4)}{s^2(s+2)^2}$$

Asymptotes: ***K* > 0:** $60^o, \quad 180^o, \quad 300^o$ ***K* < 0:** $0^o, \quad 120^o, \quad 240^o$

Intersect of Asymptotes:

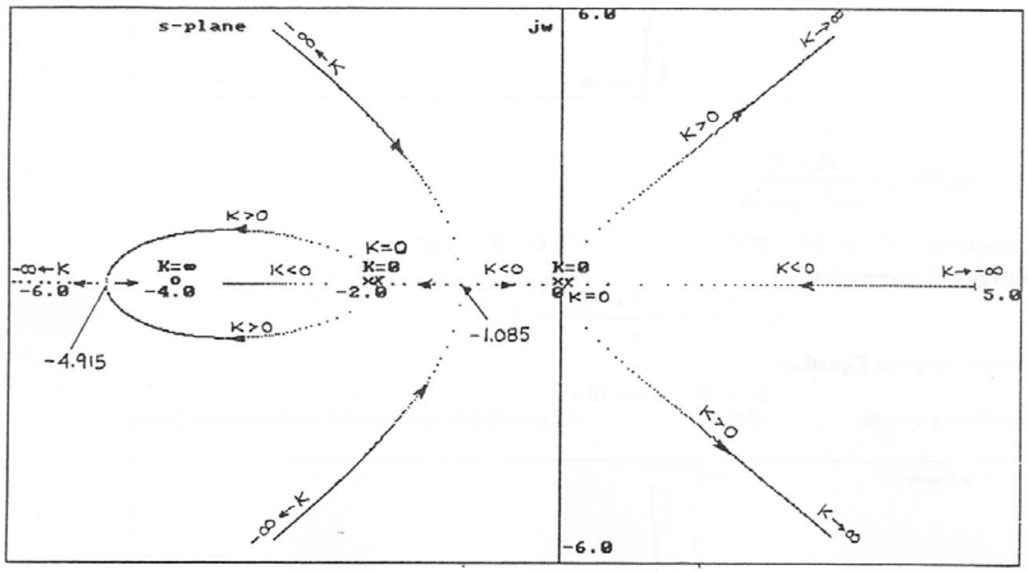
$$\sigma_1 = \frac{0+0-2-2-(-4)}{4-1} = 0$$

Breakaway-point Equation:

$$3s^4 + 24s^3 + 52s^2 + 32s = 0$$

Breakaway Points: $0, \quad -1.085, \quad -2, \quad -4.915$

Root Locus Diagram:



7-12 (d)

$$G(s)H(s) = \frac{K(s+2)}{s(s^2+2s+2)}$$

Asymptotes: ***K* > 0:** $90^o, \quad 270^o$ ***K* < 0:** $0^o, \quad 180^o$

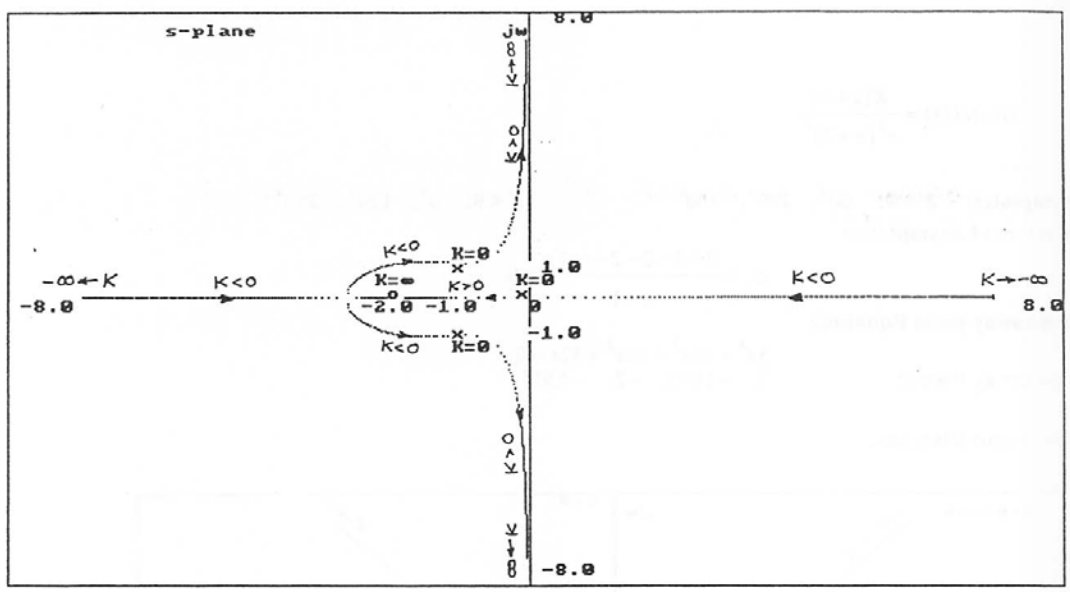
Intersect of Asymptotes:

$$\sigma_1 = \frac{0-1-j-1-j-(-2)}{3-1} = 0$$

Breakaway-point Equation: $2s^3 + 8s^2 + 8s + 4 = 0$

Breakaway Points: -2.8393 The other two solutions are not breakaway points.

Root Locus Diagram



7-12 (e)

$$G(s)H(s) = \frac{K(s+5)}{s\left(s^2+2s+2\right)}$$

Asymptotes: ***K* > 0:** 90^o, 270^o ***K* < 0:** 0^o, 180^o

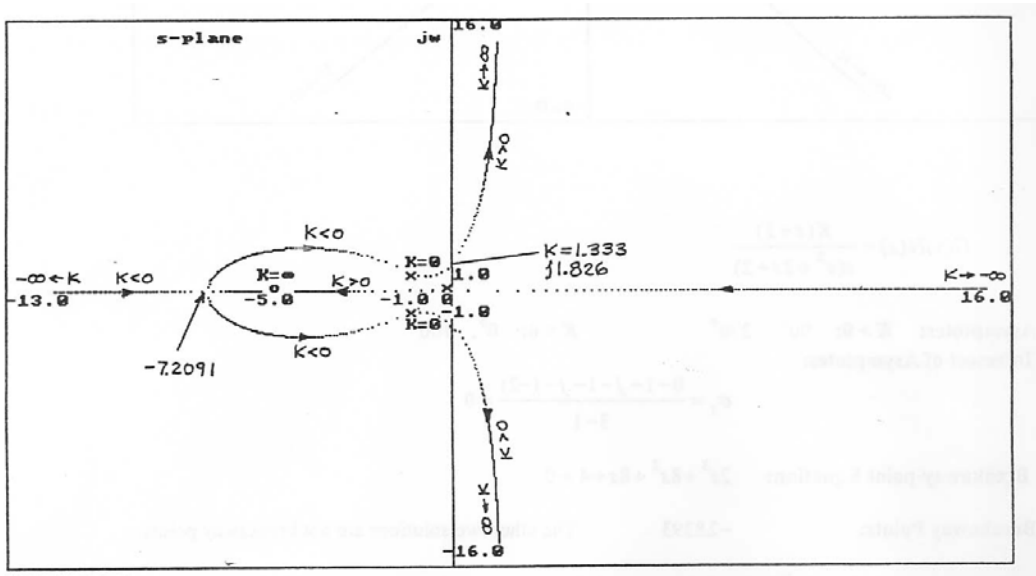
Intersect of Asymptotes:

$$\sigma_1 = \frac{0-1-j-1-j-(-5)}{3-1} = 1.5$$

Breakaway-point Equation:

$$2s^3 + 17s^2 + 20s + 10 = 0$$

Breakaway Points: -7.2091 The other two solutions are not breakaway points.



7-12 (f)

$$G(s)H(s) = \frac{K}{s(s+4)(s^2+2s+2)}$$

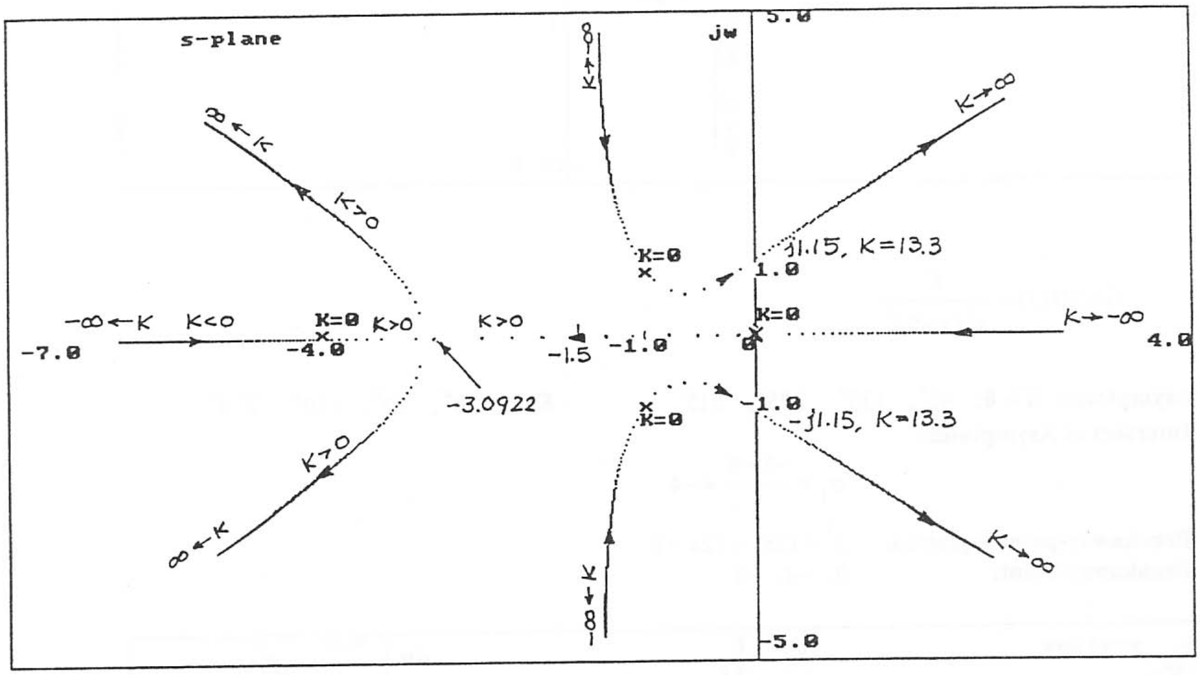
Asymptotes: ***K* > 0:** $45^o, \quad 135^o, \quad 225^o, \quad 315^o$ ***K* < 0:** $0^o, \quad 90^o, \quad 180^o, \quad 270^o$

Intersect of Asymptotes:

$$\sigma_1 = \frac{0-1-j-1+j-4}{4} = -1.5$$

Breakaway-point Equation: $4s^3 + 18s^2 + 20s + 8 = 0$

Breakaway Point: -3.0922 The other solutons are not breakaway points.



7-12 (g)

$$G(s)H(s) = \frac{K(s+4)^2}{s^2(s+8)^2}$$

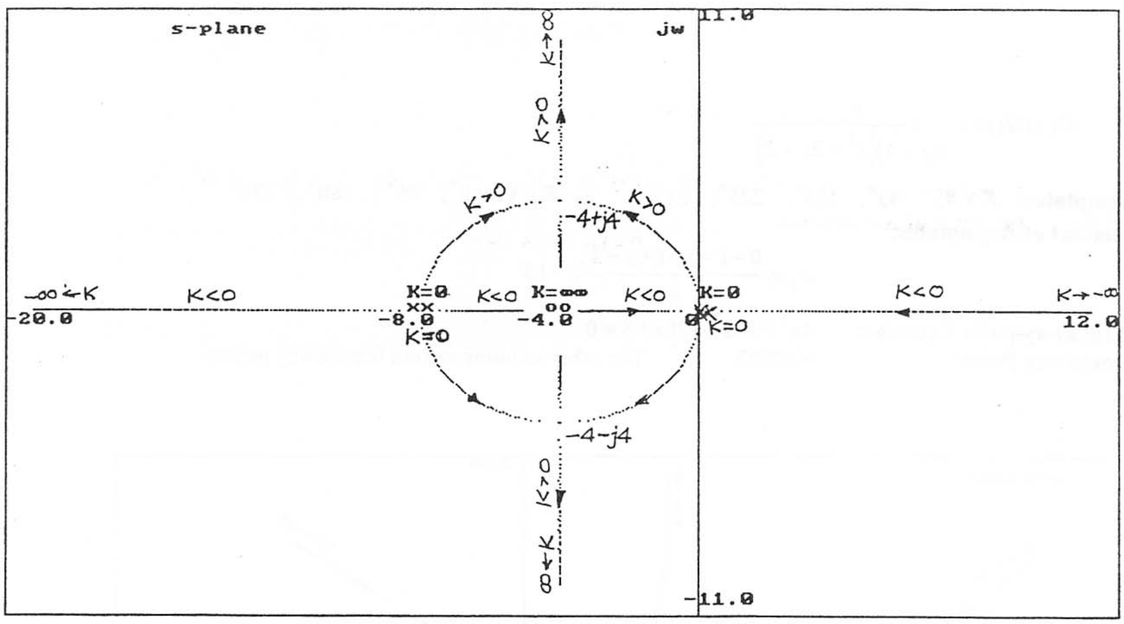
Asymptotes: ***K* > 0:** $90^o, \quad 270^o$ ***K* < 0:** $0^o, \quad 180^o$

Intesect of Asymptotes:

$$\sigma_1 = \frac{0+0-8-8-(-4)-(-4)}{4-2}$$

Breakaway-point Equation: $s^5 + 20s^4 + 160s^3 + 640s^2 + 1040s = 0$

Breakaway Points: 0, −4, −8, −4 − *j*4, −4 + *j*4



7-12 (h)

$$G(s)H(s) = \frac{K}{s^2(s+8)^2}$$

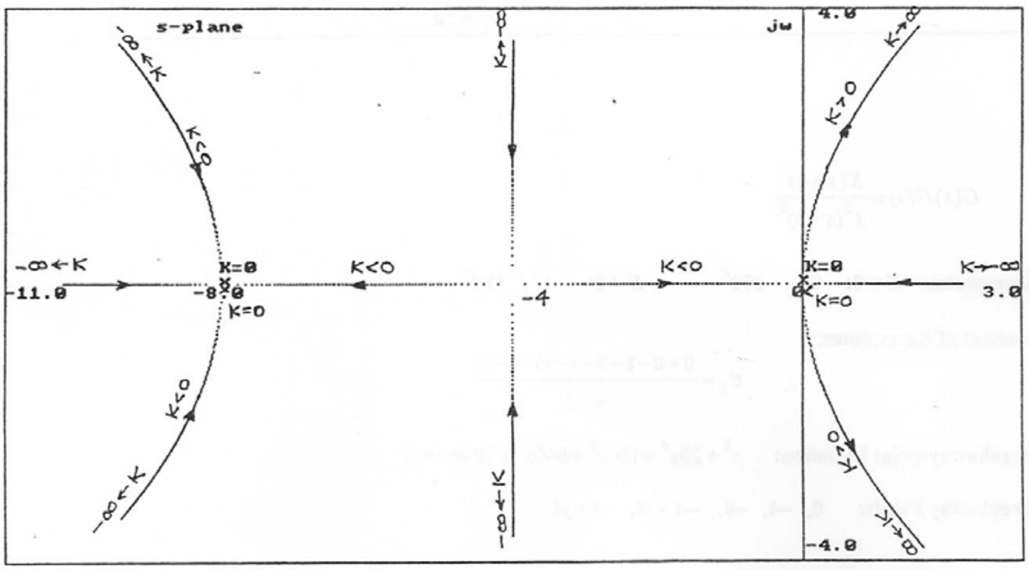
Asymptotes: ***K* > 0:** $45^o, \quad 135^o, \quad 225^o, \quad 315^o$ ***K* < 0:** $0^o, \quad 90^o, \quad 180^o, \quad 270^o$

Intersect of Asymptotes:

$$\sigma_1 = \frac{-8-8}{4} = -4$$

Breakaway-point Equation: $s^3 + 12s^2 + 32s = 0$

Breakaway Point: 0, −4, −8



7-12 (i)

$$G(s)H(s) = \frac{K\left(s^2 + 8s + 20\right)}{s^2 (s+8)^2}$$

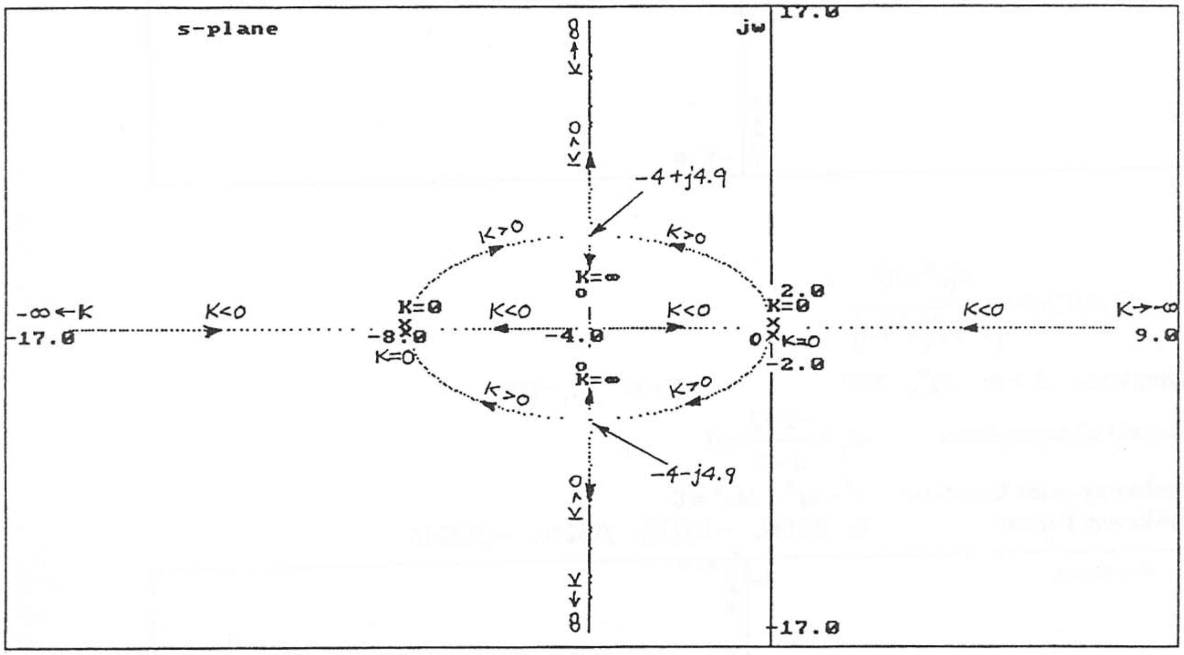
Asymptotes: ***K* > 0:** $90^o, \quad 270^o$ ***K* < 0:** $0^o, \quad 180^o$

Intersect of Asymptotes:

$$\sigma_1 = \frac{-8-8-(-4)-(-4)}{4-2} = -4$$

Breakaway-point Equation: $s^5 + 20s^4 + 128s^3 + 736s^2 + 1280s = 0$

Breakaway Points: $-4, \quad -8, \quad -4 + j4.9, \quad -4 - j4.9$



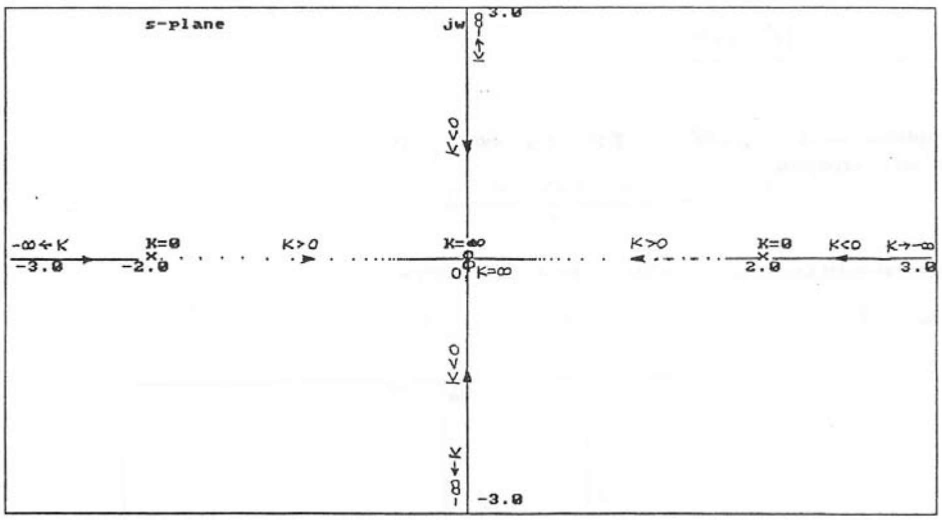
(j)

$$G(s)H(s) = \frac{Ks^2}{\left(s^2 - 4\right)}$$

Since the number of finite poles and zeros of $G(s)H(s)$ are the same, there are no asymptotes.

Breakaway-point Equation: $8s = 0$

Breakaway Points: $s = 0$



7-12 (k)

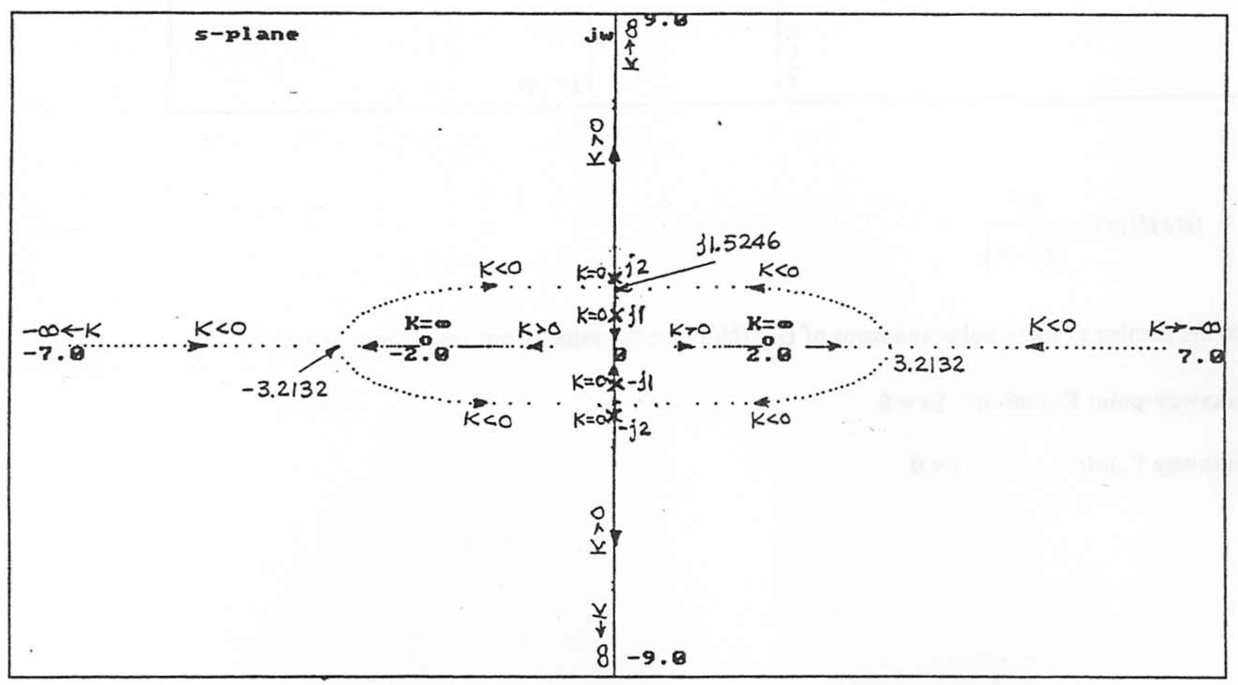
$$G(s)H(s) = \frac{K\left(s^2 - 4\right)}{\left(s^2 + 1\right)\left(s^2 + 4\right)}$$

Asymptotes: ***K* > 0:** $90^o, \quad 270^o$ ***K* < 0:** $0^o, \quad 180^o$

Intersect of Asymptotes: $\sigma_1 = \dfrac{-2+2}{4-2} = 0$

Breakaway-point Equation: $s^6 - 8s^4 - 24s^2 = 0$

Breakaway Points: $0, \quad 3.2132, \quad -3.2132, \quad j1.5246, \quad -j1.5246$



7-12 (l)

$$G(s)H(s) = \frac{K(s^2-1)}{\left(s^2+1\right)\left(s^2+4\right)}$$

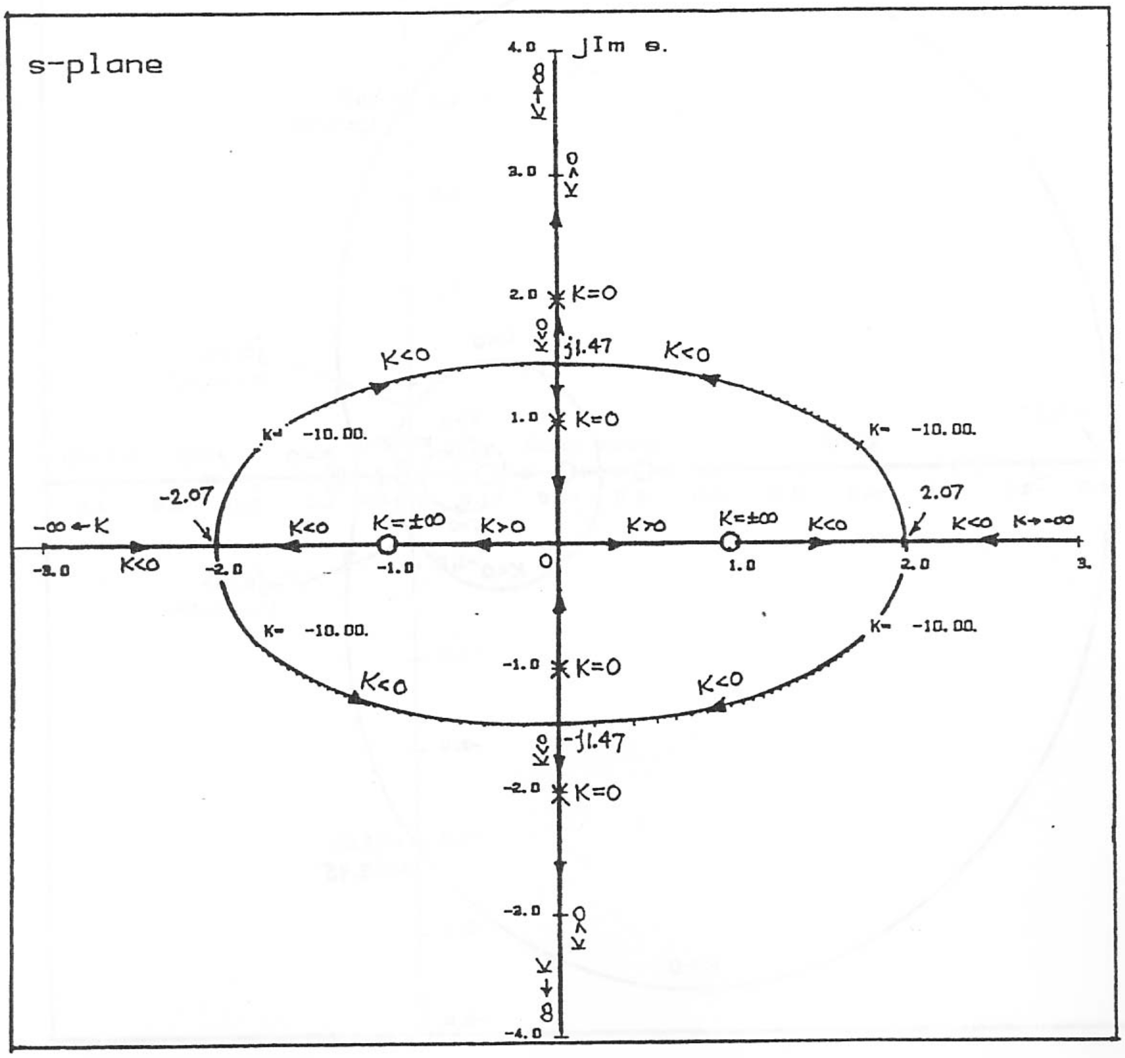
Asymptotes: *K* > 0: $90^o, \quad 270^o$ ***K* < 0:** $0^o, \quad 180^o$

Intersect of Asymptotes:

$$\sigma_1 = \frac{-1+1}{4-2} = 0$$

Breakaway-point Equation: $s^5 - 2s^3 - 9s = 0$

Breakaway Points: $-2.07, \quad 2.07, \quad -j1.47, \quad j1.47$



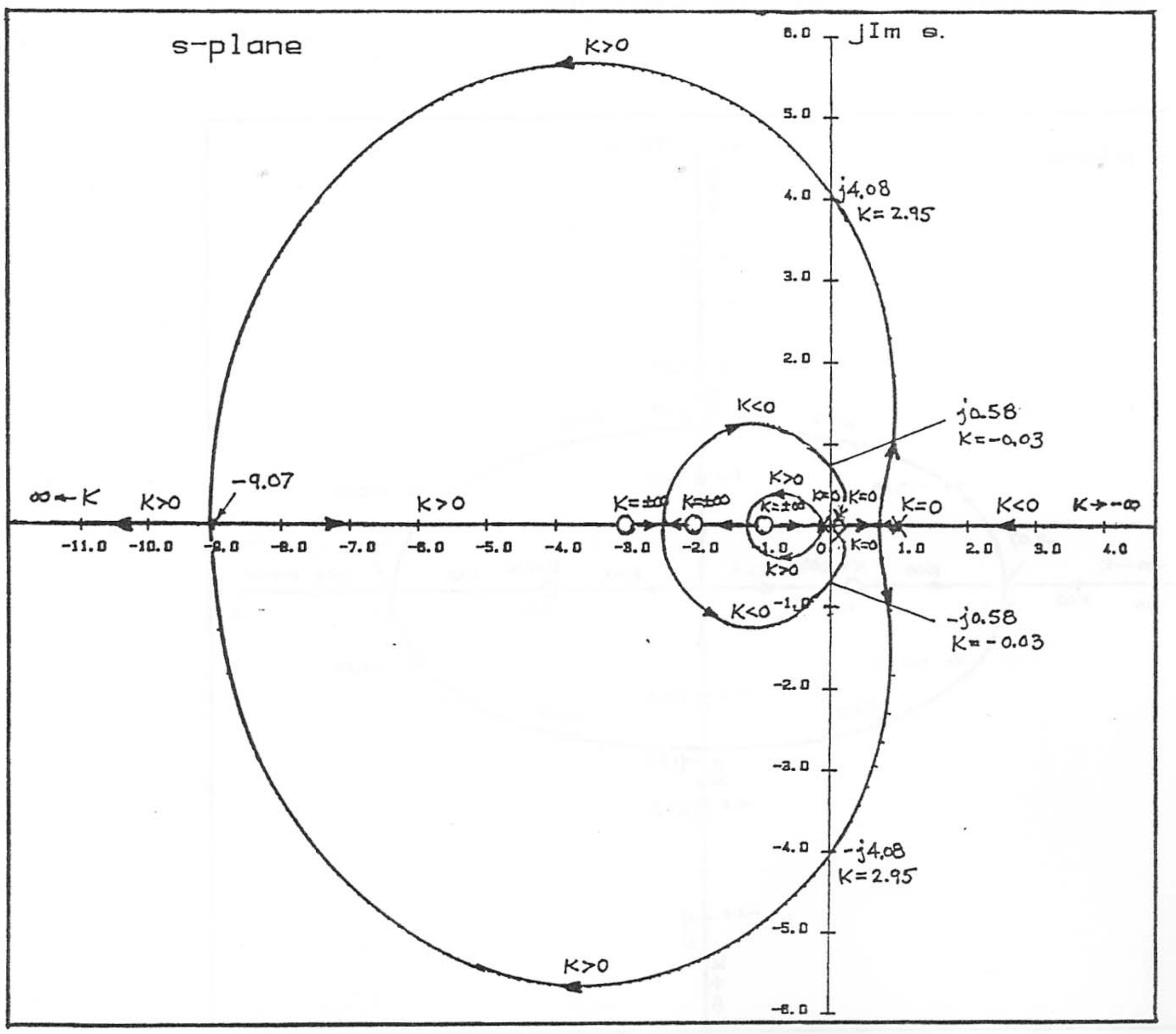
(m)

$$G(s)H(s) = \frac{K(s+1)(s+2)(s+3)}{s^3(s-1)}$$

Asymptotes: ***K* > 0:** 180^o ***K* < 0:** 0^o

Breakaway-point Equation: $s^6 + 12s^5 + 27s^4 + 2s^3 - 18s^2 = 0$

Breakaway Points: –1.21, –2.4, –9.07, 0.683, 0, 0



(n)

$$G(s)H(s) = \frac{K(s+5)(s+40)}{s^3(s+250)(s+1000)}$$

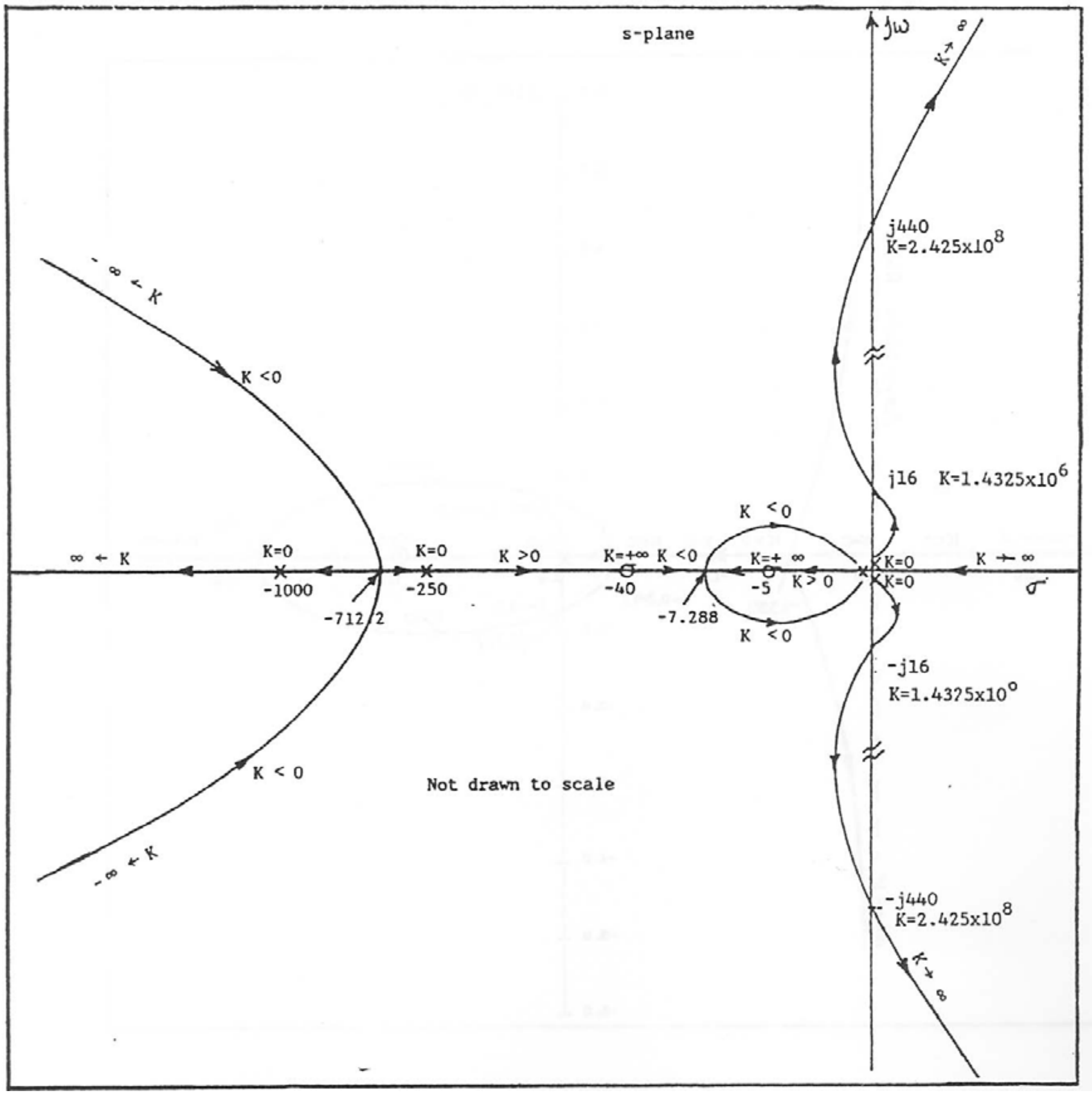
Asymptotes: ***K* > 0:** $60^o, \quad 180^o, \quad 300^o$ ***K* < 0:** $0^o, \quad 120^o, \quad 240^o$

Intersect of asymptotes:

$$\sigma_1 = \frac{0+0+0-250-1000-(-5)-(-40)}{5-2} = -401.67$$

Breakaway-point Equation: $3750s^6 + 335000s^5 + 5.247\times10^8 s^4 + 2.9375\times10^{10} s^3 + 1.875\times10^{11} s^2 = 0$

Breakaway Points: –7.288, –712.2, 0, 0



7-12 (o)

$$G(s)H(s) = \frac{K(s-1)}{s(s+1)(s+2)}$$

Asymptotes: ***K* > 0:** $90^o, \quad 270^o$ ***K* < 0:** $0^o, \quad 180^o$

Intersect of Asymptotes:

$$\sigma_1 = \frac{-1-2-1}{3-1} = -2$$

Breakaway-point Equation: $s^3 - 3s - 1 = 0$

Breakaway Points; –0.3473, –1.532, 1.879