

THE HEAT TRANSFER EFFECTS OF TUBE WALL ON A PARALLEL-LOOP THERMOSYPHON

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ABSTRACT

The effects of heat transfer of tube wall on the nature convection flows in a parallel loops thermosyphon are investigated analytically. The results show that the thermosyphon with consideration of those effects is more stable than that without the consideration. They also reveal that an increase the value of thermal diffusivity ratio, α or radius ratio, δ will increase the magnitude of the onset critical Rayleigh number, Ra_c and reduce the velocity of steady flow.

Introduction

Natural convection loops (thermosyphons) of single phase fluid have many applications, most of them related to the transport of heat from one plane to another without the use of external mechanical energy sources. For example, engineering applications include the removal of decay heat generated by a shut-down nuclear reactor core, solar systems, cooling of turbine blades and electronic devices, and growth of protein crystals [1,4]. Reviews on natural convection loops have been written in the last decade [5,6]. Stability, bifurcation and chaos features make problems of natural convection loops interesting from the academic point of view and careful system design is needed in practical applications. Extensive work has been done on these topics [7,10].

It should be pointed out that previous studies all neglect the heat transfer effects of tube wall. But Barozzi [11] and Faghri and [12] have show, in the system of conjugate heat transfer in a pipe

with internal laminar fluid flow, that the effect of wall axial conduction can readily overwhelm the effect of fluid axial conduction. Since, in a general point of view, a thermosyphon system is also a system with conjugate heat transfer. The heat transfer effect, especially the effect of axial conduction, of the tube wall in a thermosyphon system may be very important and unnegligible.

This study is concerned with the onset of motion and steady-state flows in a parallel loops thermosyphon with the effects of heat transfer of the tube wall. The same problem without heat transfer effects of tube wall have been studied by Zvirin [13]. He showed that the onset critical Rayleigh number, Ra_c , is equal to 6. The main results of this study are that an increase of the value of thermal diffusivity ratio, α , or radius ratio, δ will increase the magnitude of Ra_c or reduce the velocity of the steady flow. And if $\alpha = 0$ or $\delta = 1$ then the result will equal to that of Zvirin [13].

One Dimensional Model

The one-dimensional model analyzed here follows the methodology developed in the previous studies. The form of the loop is shown in Fig. 1. It consists of two vertical insulated tubes connecting two rigid containers. The system is filled with working fluid whose temperature is maintained at constant value T_D and T_U in the lower and upper containers separately, and heating from below is investigated, i.e. $T_D > T_U$. It is assumed that fluid properties are constant except that the density variation obeys the Boussinesq assumption with $\rho = \rho_0 [1 - \gamma(T - T_0)]$. Where ρ_0 is the density at the reference temperature T_0 and for convenience, we choose $T_D = T_0$. Then the one dimensional continuous, momentum, and energy equation of this system can be written as

$$V = V(\tau) \quad (1)$$

$$\frac{\partial V}{\partial \tau} = \frac{\gamma g}{2h} \int_0^h (T_f^r - T_f^l) dZ - \frac{8\nu V}{a} \quad (2)$$

$$\frac{\partial T_f}{\partial \tau} \pm V \frac{\partial T_f}{\partial Z} = \alpha_f^* \frac{\partial^2 T_f}{\partial Z^2} - \frac{2h_i}{\rho_f c_f a} (T_f - T_w) \quad \begin{matrix} +r \\ -l \end{matrix} \quad (3)$$

$$\frac{\partial T_w}{\partial \tau} = \alpha_w^* \frac{\partial^2 T_w}{\partial Z^2} - \frac{2a}{b^2 - a^2} \frac{2h_i}{\rho_w c_w} (T_f - T_w) \quad (4)$$

The boundary conditions for the energy equations (3) and (4) are determined by the container's temperature as shown below :

$$T_w = T_f = T_D \quad \text{at} \quad Z=0, \quad T_w = T_f = T_U \quad \text{at} \quad Z = h \quad (5a,b)$$

Equations (2)-(4) and boundary conditions (5a,b) can be transformed to the following dimensionless form :

$$\frac{1}{P} \frac{\partial v}{\partial t} = Ra \int_0^1 (\Theta^r - \Theta^l) dz - v \quad (6)$$

$$\frac{\partial \Theta}{\partial t} \pm v \frac{\partial \Theta}{\partial z} = \frac{\partial^2 \Theta}{\partial z^2} - N (\Theta - \Phi) \quad \begin{matrix} +r \\ -l \end{matrix} \quad (7)$$

$$\frac{\partial \Phi}{\partial t} = \alpha \frac{\partial^2 \Phi}{\partial z^2} - \frac{\beta}{\delta^2 - 1} \frac{2h_i}{\rho_w c_w} N (\Theta - \Phi) \quad (8)$$

$$\Theta = \Phi = 1 \quad \text{at} \quad z = 0 \quad \Theta = \Phi = 0 \quad \text{at} \quad z = 1 \quad (9a,b)$$

where the non-dimensional length, time, velocity and temperature are defined by

$$z = \frac{Z}{h}, \quad t = \frac{\tau \cdot \alpha_f^*}{h^2}, \quad v = \frac{V \cdot h}{\alpha_f^*}, \quad \Theta = \frac{T_f - T_u}{T_0 - T_u}, \quad \Phi = \frac{T_w - T_u}{T_0 - T_u} \quad (10)$$

The parameters appearing in equation (6)-(8) are the Rayleigh number, thermal diffusivity ratio, radius ratio, heat capacity ratio, and the modified Nusselt number :

$$Ra = \frac{\gamma g (T_0 - T_u) a^2 h}{16 \nu \alpha_f^*}, \quad P = 8 \frac{P r}{s_i^2}, \quad \alpha = \frac{\alpha_w^*}{\alpha_f^*}, \quad \delta = \frac{b}{a}, \quad \beta = \frac{\rho_f c_f}{\rho_w c_w}, \quad N = \frac{Nu}{s_i^2} = \frac{2h_i \cdot a}{s_i^2 \cdot k_f} \quad (11)$$

where $s_i = a/h$ is the shape ratio, Pr is the Prandtl number, and Nu is the Nusselt number.

The Onset of Motion-Stability of the Rest State

In the previous study [13] it have been shown that no steady flow can be established in the loop when the Rayleigh number, Ra , is below a critical value, Ra_c . Now we first investigate the instability associated with the onset of motion in the loop. The rest basic state is governed by equations (6)-(9) with $\partial/\partial t = 0$ and $\bar{v} = 0$. The conductive solutions in this case are :

$$\bar{\Theta} = \bar{\Phi} = 1 - z \quad (12)$$

Linearized stability analysis is used to study the stability of rest state. The time dependent velocity and temperature can be expressed by

$$v = v' e^{\sigma t}, \quad \Theta = \bar{\Theta} + \Theta'(z) e^{\sigma t}, \quad \Phi = \bar{\Phi} + \Phi'(z) e^{\sigma t} \quad (13a,b,c)$$

Where $\bar{\Theta}$ and $\bar{\Phi}$ are the basic solutions and $v' e^{\sigma t}$ et al. are the small perturbations. σ is the characteristic value of this dynamic system. In general, σ may be complex and the basic state is unstable when $\text{Real}(\sigma) > 0$.

Introduction of these expressions into momentum and energy equations (6)-(8) yields:

$$v' (1 + \frac{\sigma}{P}) = Ra \int_0^1 (\Theta' - \Phi') dz \quad (14)$$

$$\sigma\Theta \pm v' \frac{\partial \bar{\Theta}}{\partial z} = \frac{\partial^2 \bar{\Theta}}{\partial z^2} - N(\Theta - \Phi') \quad \begin{matrix} +r \\ -l \end{matrix} \quad (15)$$

$$\sigma\Phi' = \alpha \frac{\partial^2 \Phi'}{\partial z^2} + \frac{\beta}{\delta^2 - 1} N(\Theta - \Phi') \quad (16)$$

$$\text{with the boundary conditions :} \quad \Theta' = \Phi' = 0 \quad \text{at} \quad z = 0, 1 \quad (17)$$

The perturbed energy equations (15) and (16) with the boundary condition (17) are first solved for Θ' and Φ' in terms of v' as an unknown constant. Then by introducing Θ' into equation (14) followed by an integration yields the characteristic equation for the stability parameter as :

$$1 + \frac{\sigma}{P} = 2Ra \left[\sum_{i=1}^4 \frac{A_i}{\zeta_i} (e^{\zeta_i} - 1) - A_5 \right] \quad (18)$$

where the coefficients A_i , ζ_i , and A_5 can be found in the Appendix.

Note that the onset motion from the rest state to the steady convective flow corresponding to the monotonic instability of this system. It means that the characteristic value, σ , is real and the unstable disturbance grows exponentially with time. By letting $\sigma = 0$ in equation (18) one can obtain the corresponding critical value of Ra in terms of parameters α , β , δ and N .

Steady-State Solution

We now seek the solutions of equations (6)-(8) for the steady state. If $Ra < Ra_c$ the steady state solutions are corresponding to the conductive solutions. While for $Ra > Ra_c$ the steady state solutions are corresponding to the convective flow of the working fluid. In the steady state the time derivative of the governing equation will disappear and the velocity takes on a constant value, $\bar{v}$. Then the systems (6)-(9) become to the following form :

$$\bar{v} = Ra \int_0^1 (\bar{\Theta}' - \bar{\Phi}') dz \quad (19)$$

$$\pm \bar{v} \frac{\partial \bar{\Theta}}{\partial z} = \frac{\partial^2 \bar{\Theta}}{\partial z^2} - N(\bar{\Theta} - \bar{\Phi}) \quad \begin{matrix} +r \\ -l \end{matrix} \quad (20)$$

$$\alpha \frac{\partial^2 \bar{\Phi}}{\partial z^2} + \frac{\beta}{\delta^2 - 1} N(\bar{\Theta} - \bar{\Phi}) = 0 \quad (21)$$

$$\bar{\Theta} = \bar{\Phi} = 1 \quad \text{at} \quad z = 0, \quad \bar{\Theta} = \bar{\Phi} = 1 \quad \text{at} \quad z = 1 \quad (22a,b)$$

The solution procedure is similar to that of the stability problem. The final results of $\bar{\Theta}$, $\bar{\Phi}$, and $\bar{v}$ in terms of parameters α , β , δ , Ra and N are expressed in the following form :

$$\overline{\Theta}^r = \sum_{i=1}^3 B_i e^{\lambda_i z} + B_4, \quad \overline{\Theta}^l = \sum_{i=1}^3 C_i e^{\xi_i z} + C_4, \quad \overline{\Phi}^r = \sum_{i=1}^3 B_i b_i e^{\lambda_i z} + B_4, \quad \overline{\Phi}^l = \sum_{i=1}^3 C_i g_i e^{\xi_i z} + C_4. \quad (23a,b,c,d)$$

$$\bar{v} = Ra \left\{ \left[\sum_{i=1}^3 \frac{B_i}{\lambda_i} (e^{\lambda_i} - 1) + B_4 \right] - \left[\sum_{i=1}^3 \frac{C_i}{\xi_i} (e^{\xi_i} - 1) + C_4 \right] \right\} \quad (23e)$$

where the coefficient B_i and C_i can be found in the Appendix. Note that if the parameters are given then from equations (23a,e) one can obtain the steady convective solutions of $\bar{v}$ and $\overline{\Theta}$ of the working fluid and the corresponding conjugate solution of $\overline{\Phi}$ of the tube wall.

Results and Discussions

The method to derive the steady solutions of a thermosyphon system and their exchange of stability are described in above sections. Here we will show the results and discuss their characteristics. Fig. 2 shows that the onset critical Rayleigh number, Ra_c , will increase monotonically with increasing the magnitude of thermal diffusivity ratio, α , or radius ratio, δ . And when $\alpha \rightarrow 0$ or $\delta \rightarrow 1$ the value of Ra_c will approach to 6 which is identical to that of [13]. Fig. 3 indicates that increasing the thermal diffusivity ratio will reduce the velocity of steady flow. Also, this figure shows that the critical point, $Ra = Ra_c$, is a pitchfork bifurcation point. If $Ra < Ra_c$ only conductive solution is possible ($\bar{v} = 0$). But if $Ra > Ra_c$ the conductive solution will become unstable and the solution will evolve monotonically into the convective one ($\bar{v} > 0$). This dynamic behavior near the critical condition is denoted as the "principle of exchange of stability". Fig. 4 shows that the steady velocity will decrease with increasing the value of δ .

The above results all indicate that the parallel loops thermosyphon system with consideration of the heat transfer effect of tube wall is more stable than that without the consideration. This is because that parts of energy are carried by the axial conduction of tube wall which results in a reduction of convective heat transfer of working fluid. At the same time the buoyancy effect of working fluid will decrease and the flow system naturally become more stable. And this stable tendency will increase axial conduction of tube wall, that is, increasing the magnitude of α or δ . The figures of temperature distributions can explain the stable mechanism more clearly. Fig. 5a,b and 6a,b show the temperature distributions, Θ and Φ , in the right and left branch respectively. It is shown in the right branch (Fig. 5a or 6a), that working fluid will heat the tube wall and this heating process will reduce the temperature of working fluid. On the other hand, in the left branch (Fig. 5b or 6b) its behavior is reverse which results in raising the temperature of working fluid. Also, the tendency of reducing (raising) the temperature of working fluid in the right (left) branch will be increased with increasing the value of α or δ . Furthermore, from the first term at the right hand of equation (6), it is known that in Fig. 7 the area enclosed by the temperature distributions of working fluid of right and left branches is proportional to the buoyancy force of the thermosyphon system. It is found that the area is reduced when the heat transfer effects of tube wall are

considered. It means that the effects of buoyancy are reduced. Which can explain why the parallel loops thermosyphon with the effects of heat transfer of tube wall is more stable than that without the effects.

Conclusion

The onset stability and steady flow in a parallel loops thermosyphon system with the heat transfer effects of tube wall are investigated analytically. The results show that the buoyancy force in the working fluid will reduce when those effects are considered. Which results in stabilizing the thermosyphon system, that is an increase of the value of onset critical Rayleigh number or a reduction of the velocity of steady flow. And this stabilized effects will increase with increasing axial conductive effects of tube wall, i.e., increasing the magnitudes of thermal diffusivity ratio, α , or radius ratio, δ .

Nomenclature

A_i	constant coefficient, Appendix	P	modified Prantdl number
a	inner radius of tube	Pr	Prandtl number = α / ν
B_i	constant coefficient, Appendix	Ra	Rayleigh number
b	outer radius of tube	Ra_c	onset critical Rayleigh number
C_i	constant coefficient, Appendix	s_i	shape ratio = a/h
c	specific heat capacity	T	temperature
g	gravity	t	dimensionless time
h	hight of loop	V	velocity
h_i	convective heat transfer coefficient	ν	dimensionless velocity
N	modified Nusselt number = Nu/s_i^2	Z	vertical coordinate
Nu	Nusselt number	z	dimensionless vertical coordinate

Greek symbols

α	thermal diffusivity ratio	ν	viscosity of fluid
α^*	thermal diffusivity	ξ_i	constant coefficient, Appendix
β	thermal expansion coefficient	ρ	density
γ	heat capacity ratio	σ	characteristic value
δ	radius ratio	τ	time
ζ_i	constant coefficient, Appendix	Θ	dimensionless temperature of fluid
λ_i	constant coefficient, Appendix	Φ	dimensionless temperature of tube wall

Subscripts

D	lower container
f	working fluid

Superscripts

l	left loop
r	right loop

U	upper container	-	steady state
w	tube wall	'	perturbed state

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Appendix

The constant A_i 's, B_i 's, ζ_i 's, λ_i 's, ξ_i 's, b_i 's and g_i 's presented in equation (18) , (23) can be determined from the following algebra equations and expressions :

$$\sum_{j=1}^4 D_{ij} A_j = E_i \quad \text{for} \quad i = 1 \dots 4, \quad A_5 = \frac{a_2}{a_3} \quad (\text{A-1a,b})$$

$$\zeta_1 = \sqrt{\frac{a_2 + \sqrt{a_2^2 - 4a_1a_3}}{2}}, \quad \zeta_2 = \sqrt{\frac{a_2 - \sqrt{a_2^2 - 4a_1a_3}}{2}}, \quad \zeta_3 = -\zeta_1, \quad \zeta_4 = -\zeta_2 \quad (\text{A-2a,b,c,d})$$

$$E_1 = \frac{a_4}{a_3}, \quad E_2 = \frac{a_4}{a_3}, \quad E_3 = \left(1 + \frac{\sigma}{N}\right) \frac{a_4}{a_3} + \frac{1}{N}, \quad E_4 = \left(1 + \frac{\sigma}{N}\right) \frac{a_4}{a_3} + \frac{1}{N} \quad (\text{A-3})$$

$$\sum_{j=1}^4 F_{ij} B_j = G_i \quad \text{for} \quad i = 1 \dots 4 \quad (\text{A-4})$$

$$d_1 \lambda^3 + d_2 \lambda^2 + d_3 \lambda + d_4 = (\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) = 0 \quad (\text{A-5})$$

$$\sum_{j=1}^4 H_{ij} C_j = K_i \quad \text{for} \quad i = 1 \dots 4 \quad (\text{A-6})$$

$$b_1 = 1 + \frac{\beta \bar{v}}{N} - \frac{\lambda_1^2}{N}, \quad b_2 = 1 + \frac{\beta \bar{v}}{N} - \frac{\lambda_2^2}{N}, \quad b_3 = 1 + \frac{\beta \bar{v}}{N} - \frac{\lambda_3^2}{N} \quad (\text{A-7a,b,c})$$

$$d_1 \xi^3 + d_2 \xi^2 + d_3 \xi + d_4 = (\xi - \xi_1)(\xi - \xi_2)(\xi - \xi_3) = 0 \quad (\text{A-8})$$

$$g_1 = 1 + \frac{\beta \bar{v}}{N} - \frac{\xi_1^2}{N}, \quad g_2 = 1 + \frac{\beta \bar{v}}{N} - \frac{\xi_2^2}{N}, \quad g_3 = 1 + \frac{\beta \bar{v}}{N} - \frac{\xi_3^2}{N} \quad (\text{A-9a,b,c})$$

$$\text{where } D_{11} = D_{12} = D_{13} = D_{14} = 1, \quad D_{21} = e^{\xi_1}, \quad D_{22} = e^{\xi_2}, \quad D_{23} = e^{\xi_3}, \quad D_{24} = e^{\xi_4},$$

$$D_{31} = 1 + \frac{\sigma}{N} - \frac{\xi_1^2}{N}, \quad D_{32} = 1 + \frac{\sigma}{N} - \frac{\xi_2^2}{N}, \quad D_{33} = 1 + \frac{\sigma}{N} - \frac{\xi_3^2}{N}, \quad D_{34} = 1 + \frac{\sigma}{N} - \frac{\xi_4^2}{N},$$

$$D_{41} = D_{31} e^{\xi_1}, \quad D_{42} = D_{32} e^{\xi_2}, \quad D_{43} = D_{33} e^{\xi_3}, \quad D_{44} = D_{34} e^{\xi_4},$$

$$a_1 = -\frac{\alpha}{N}, \quad a_2 = \frac{\sigma}{N} + \alpha \left(1 + \frac{\sigma}{N}\right) + \frac{\beta}{(\delta^2 - 1)}, \quad a_3 = -\alpha \left(1 + \frac{\sigma}{N}\right) - \frac{\beta \cdot \sigma}{(\delta^2 - 1)}, \quad a_4 = \frac{\sigma}{N} + \frac{\beta}{(\delta^2 - 1)},$$

$$E_1 = \frac{a_4}{a_3}, \quad E_2 = \frac{a_4}{a_3}, \quad E_3 = \left(1 + \frac{\sigma}{N}\right) \frac{a_4}{a_3} + \frac{1}{N}, \quad E_4 = \left(1 + \frac{\sigma}{N}\right) \frac{a_4}{a_3} + \frac{1}{N},$$

$$F_{11} = F_{12} = F_{13} = F_{14} = 1, \quad F_{21} = e^{\lambda_1}, \quad F_{22} = e^{\lambda_2}, \quad F_{23} = e^{\lambda_3}, \quad F_{24} = e^{\lambda_4},$$

$$F_{31} = b_1, \quad F_{32} = b_2, \quad F_{33} = b_3, \quad F_{34} = 1, \quad F_{41} = b_1 e^{\lambda_1}, \quad F_{42} = b_2 e^{\lambda_2}, \quad F_{43} = b_3 e^{\lambda_3}, \quad F_{44} = 1,$$

$$d_1 = -\frac{\alpha}{N}, \quad d_2 = \frac{\alpha \bar{v}}{N}, \quad d_3 = \alpha + \frac{\beta}{\delta^2 - 1}, \quad d_4 = -\frac{\beta \bar{v}}{\delta^2 - 1},$$

$$G_1 = G_3 = 1, \quad G_2 = G_4 = 0, \quad K_1 = K_3 = 1, \quad K_2 = K_4 = 0,$$

$$H_{11} = H_{12} = H_{13} = H_{14} = 1, \quad H_{21} = e^{\xi_1}, \quad H_{22} = e^{\xi_2}, \quad H_{23} = e^{\xi_3}, \quad H_{24} = e^{\xi_4},$$

$$H_{31} = g_1, \quad H_{32} = g_2, \quad H_{33} = g_3, \quad H_{34} = 1, \quad H_{41} = g_1 e^{\xi_1}, \quad H_{42} = g_2 e^{\xi_2}, \quad H_{43} = g_3 e^{\xi_3}, \quad H_{44} = 1.$$

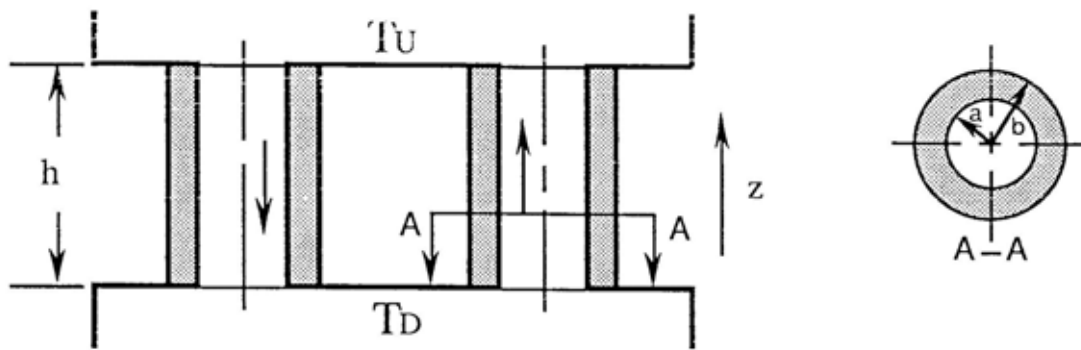


FIG. 1. A schematic description of the thermosyphon system

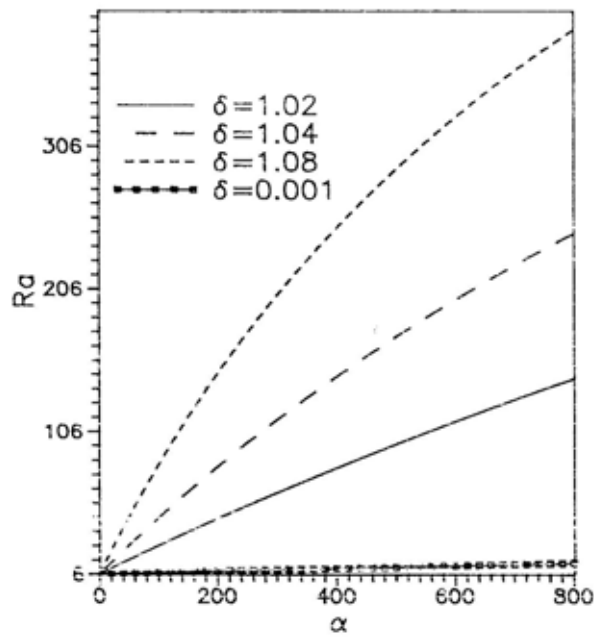


FIG. 2. Onset critical Rayleigh number, Ra_c , v.s. α with different values of δ ; $N=19200/11$, $s_l = 1/20$, $\beta = 1.2$

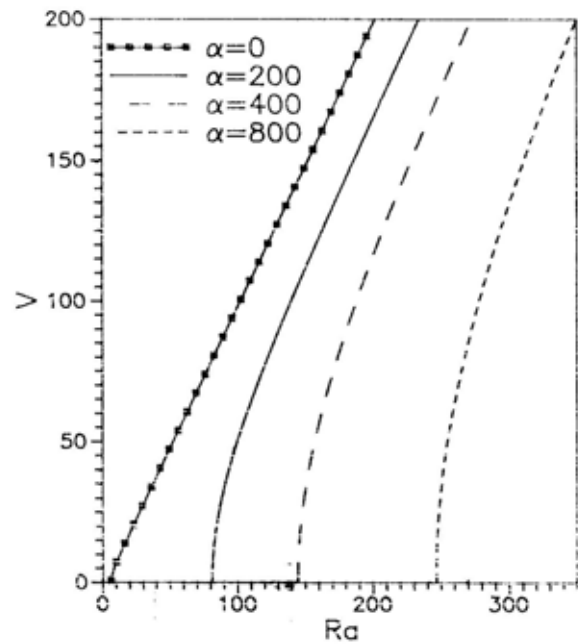


FIG. 3. Steady velocity, $\bar{v}$, v.s Ra with different values of α ; $N=19200/11$, $s_l = 1/20$, $\beta = 1.2$, $\delta = 1.04$

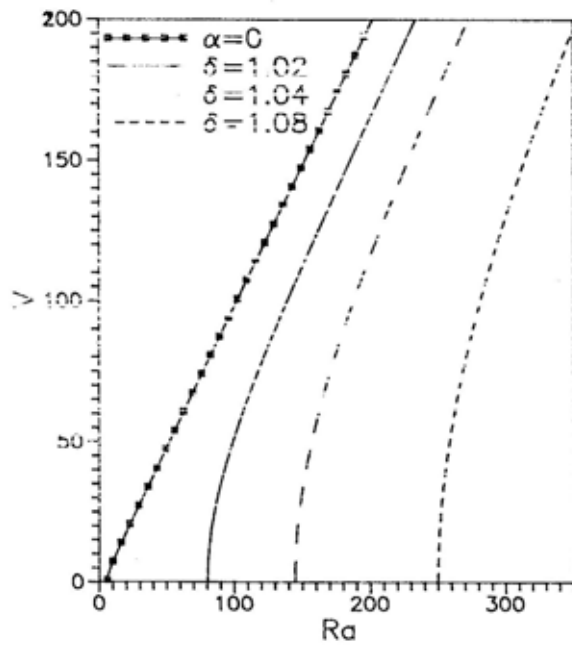


FIG. 4. Steady velocity, $\bar{v}$, v.s Ra with different values of δ ; $N=19200/11$, $s_l = 1/20$, $\alpha=400$, $\beta = 1.2$

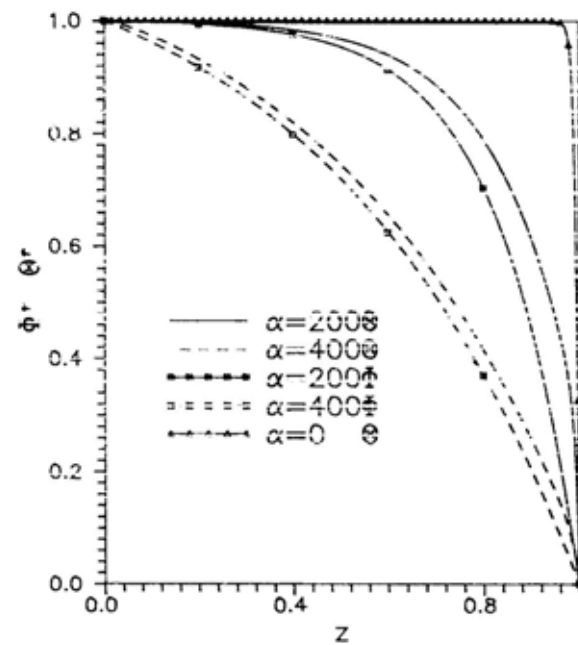


FIG. 5a. Steady temperature distribution in the right loop with different values of α ; $Ra = 160$, $N = 19200/11$, $s_l = 1/20$, $\beta = 1.2$, $\delta = 1.04$

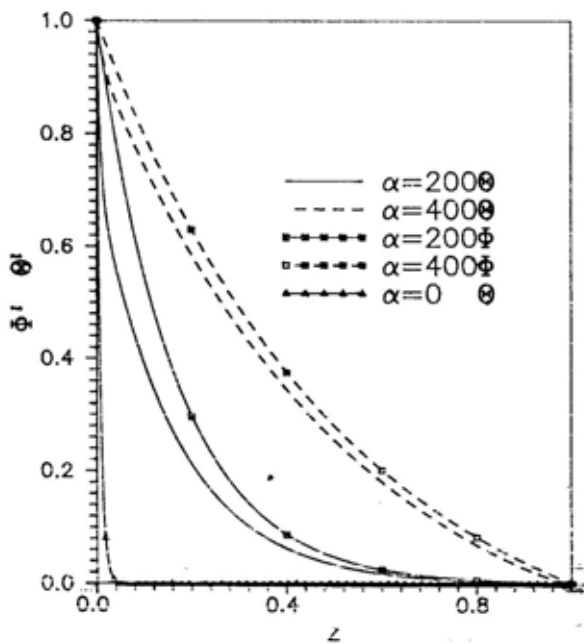


FIG. 5b. Steady temperature distribution in the left loop with different values of α ; $Ra = 160$, $N = 19200/11$, $Si = 1/20$, $\beta = 1.2$, $\delta = 1.04$

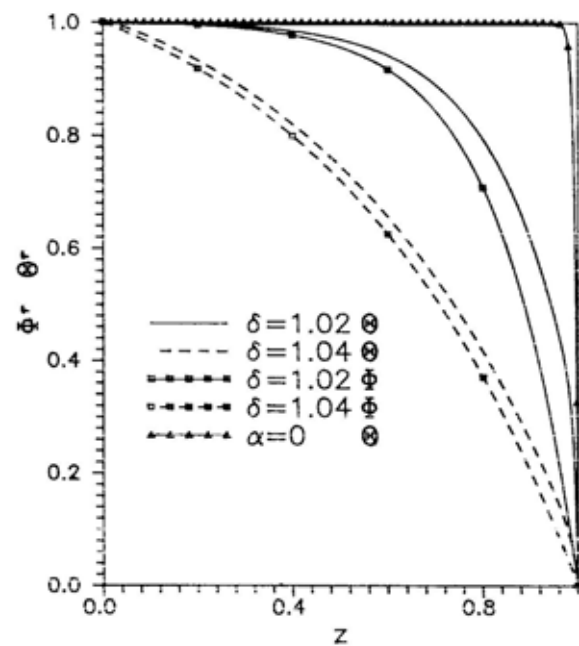


FIG. 6a. Steady temperature distribution in the right loop with different values of δ ; $Ra = 160$, $N = 19200/11$, $Si = 1/20$, $\alpha = 400$, $\beta = 1.2$

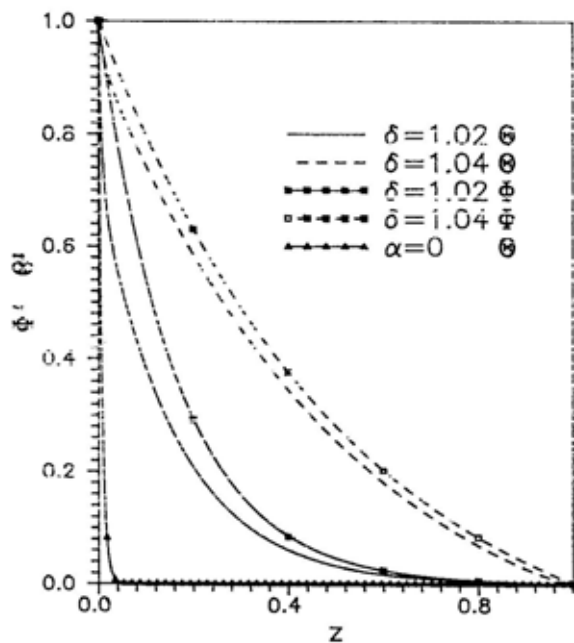


FIG. 6b. Steady temperature distribution in the left loop with different values of δ ; $Ra = 160$, $N = 19200/11$, $Si = 1/20$, $\beta = 1.2$

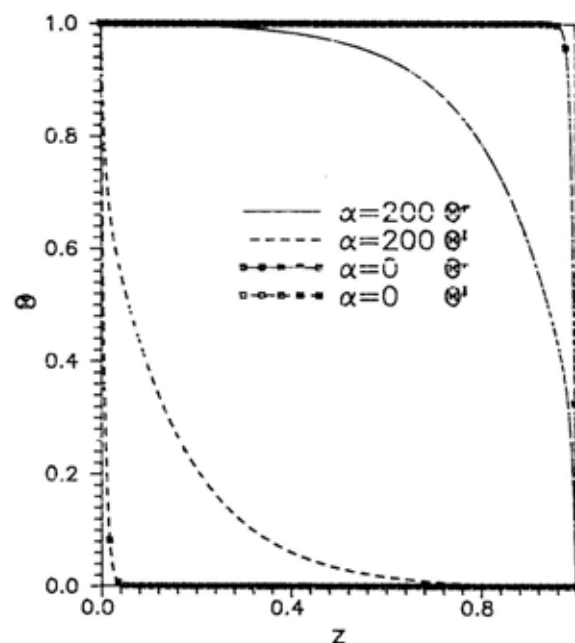


FIG. 7. Steady temperature distribution of working fluid in the right, left loops, $Ra = 160$, $N = 19200/11$, $Si = 1/20$, $\beta = 1.2$, $\delta = 1.04$