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A STUDY OF PARALLEL-LOOP THERMOSYPHON CONTAINING COLD WATER

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ABSTRACT

This paper is to investigate the onset of motion and steady-state flows of cold water in a parallel loops thermosyphon. The results show that increase the value of temperature parameter of density inversion, Θ_m will increase the magnitude of the onset critical Rayleigh number, R_{ac} , and reduce the velocity of steady flow. They also reveal that the system will always be kept in rest state when $\Theta_m > 0.5$.

Introduction

Natural circulation loops (thermosyphons) have recently gained more interest, mostly due to their applications to energy conversion systems, e.g., emergency core cooling of nuclear reactors, solar heaters, and geothermal system. Free convection loops also appear in several geophysical processes. Reviews on natural convection loops was presented by Zvirin [1] and Grief [2].

In this paper we concern the onset of motion and steady-state flows of cold water in a parallel loops thermosyphon. In the industry application, the density inversion phenomenon associated with the occurrence of the maximum density at about 4 °C will be found at the ice storage system [3], refrigeration system, heat exchanger, etc. Especially, in the thermosyphon application the PWR ice condenser containment [4] used the ice condenser compartment to make a heat sink and the reactor core to consider as the heat source.

One Dimensional Model

The one-dimensional model analyzed here follows the methodology developed in the previous studies. The form of the loop is shown in Fig. 1. It consists of two vertical insulated tubes connecting two rigid containers. The system is filled with working fluid whose temperature is maintained at constant value T_D and T_U in the lower and upper containers separately, and heating from below is investigated, i.e. $T_D > T_U$. It is assumed that fluid properties are constant except that the density variation in the buoyancy term.

The density-temperature relation of water was based on a widely used correlation proposed by Gebhart and Mollendorf [5]:

$$\rho(T) = \rho_m (1 - rsp |T - T_m|^b) \quad (1)$$

$$\text{where } \rho_m = 999.9720 \text{ kg m}^{-3}, \quad T_m = 4.29325^\circ \text{C},$$

$$b = 1.894816, \text{ and } rsp = 9.297173 \times 10^{-6} (^\circ \text{C})^{-b}$$

Then the one dimensional continuous, momentum, and energy equation of the system can be written as

$$V = V(\tau) \quad (2)$$

$$\frac{\partial V}{\partial \tau} = \frac{rsp \cdot g}{2h} \int_0^h (|T_i - T_m|^b - |T_l - T_m|^b) dZ - \frac{8\nu V}{a} \quad (3)$$

$$\frac{\partial T}{\partial \tau} \pm V \frac{\partial T}{\partial Z} = \alpha \frac{\partial^2 T}{\partial Z^2} \quad \begin{matrix} +r \\ -l \end{matrix} \quad (4)$$

The boundary conditions for the energy equations (3) and (4) are determined by the container temperature as show below:

$$T=l \quad \text{at} \quad Z=0, \quad T=0 \quad \text{at} \quad Z=h \quad (5a,b)$$

Equations (3)-(4) and boundary conditions (5a,b) can be transformed to the following dimensionless form:

$$\frac{l}{P} \frac{\partial v}{\partial t} = Ra \int_0^1 (|\Theta_i - \Theta_m|^b - |\Theta_l - \Theta_m|^b) dz - v \quad (6)$$

$$\frac{\partial \Theta}{\partial t} \pm v \frac{\partial \Theta}{\partial z} = \frac{\partial^2 \Theta}{\partial z^2} \quad \begin{matrix} +r \\ -l \end{matrix} \quad (7)$$

$$\Theta=l \quad \text{at} \quad z=0 \quad \Theta=0 \quad \text{at} \quad z=1 \quad (8a,b)$$

where the non-dimensional length, time, velocity and temperature are defined by

$$z = \frac{Z}{h}, \quad t = \frac{\tau \alpha}{h^2}, \quad v = \frac{V h}{\alpha}, \quad \Theta = \frac{T - T_U}{T_D - T_U} \quad (9)$$

The parameters appearing in equations (6)-(8) are the Rayleigh number, modified Prandtl number and the temperature parameter of density inversion.

$$Ra = \frac{g \cdot r_{sp} (T_D - T_U)^b a^2 h}{16 \nu \alpha}, \quad P = 8 Pr \frac{h^2}{a^2}, \quad \text{and} \quad \Theta_m = \frac{T_m - T_U}{T_D - T_U} \quad (10)$$

The Onset of Motion-Stability of the Rest State

In the previous study Zvirin [4] has shown that no steady flow can be established in the loop when the Rayleigh number, Ra , is below a critical value, Ra_c . Now we first investigate the instability associated with the onset of motion in the loop.

The rest basic state is governed by equations (6)-(7) with $\partial/\partial t = 0$ and $\bar{v} = 0$. The conductive solutions in this case are :

$$\bar{\Theta} = 1 - z \quad (11)$$

linearize stability analysis is used to study the stability of rest state. The velocity and temperature can be expressed by

$$v = v' e^{\sigma t}, \quad \Theta = \bar{\Theta} + \Theta'(z) e^{\sigma t} \quad (12a,b)$$

Where $\bar{\Theta}$ is the basic solution and $v' e^{\sigma t}$ and $\Theta' e^{\sigma t}$ are the small perturbations. σ is the characteristic value of this dynamic system. In general, σ may be a complex number and the basic state is unstable when $\text{Real}(\sigma) > 0$.

Introduction of these expressions into momentum and energy equations (5)(6) yields:

$$v' \left(1 + \frac{\sigma}{P}\right) = Ra \cdot b \int_0^1 \left[\text{sign}(\bar{\Theta}_r - \Theta_m) \Theta_r' |\bar{\Theta}_r - \Theta_m|^{b-1} - \text{sign}(\bar{\Theta}_l - \Theta_m) \Theta_l' |\bar{\Theta}_l - \Theta_m|^{b-1} \right] dz \quad (13)$$

$$\sigma \Theta' \pm v' \frac{\partial \bar{\Theta}}{\partial z} = \frac{\partial^2 \Theta'}{\partial z^2} \quad \begin{matrix} +r \\ -l \end{matrix} \quad (14)$$

with the boundary conditions :

$$\Theta' = 0 \quad \text{at} \quad z = 0, 1 \quad (15)$$

The perturbed energy equation (14) with the boundary conditions (15) is first solved for Θ' in terms of v' as an unknown constant. Then, by introducing Θ' into equation (13) followed by an integration yields the characteristic equation for the stability parameter as :

$$\left(1 + \frac{\sigma}{P}\right) = 2Ra \cdot b \int_0^1 \text{sign}(1 - z - \Theta_m) (C_1 e^{\sqrt{\sigma} z} + C_2 e^{-\sqrt{\sigma} z}) |1 - z - \Theta_m|^{b-1} dz \quad (16)$$

$$\text{where } C_1 = \frac{1 - e^{-\sqrt{\sigma}}}{(e^{-\sqrt{\sigma}} - e^{\sqrt{\sigma}})\sigma}, \quad C_2 = \frac{1 - e^{\sqrt{\sigma}}}{(e^{-\sqrt{\sigma}} - e^{\sqrt{\sigma}})\sigma}$$

Note that the onset motion from the rest state to the steady convective flow corresponding to the monotonic instability of this system. It means that the characteristic value, σ , is real and the unstable disturbance grows exponentially with time. By letting $\sigma = 0$ in equation (16) one can obtain the corresponding critical value of Ra in terms of Θ_m and b .

Steady-State Solution

Now we seek the solutions of equations (6)-(7) for the steady state. If $Ra < Ra_c$ the steady state solutions are corresponding to the conductive solutions. While for $Ra > Ra_c$ the steady state solutions are corresponding to the convective flow of the working fluid. In the steady state the time derivative of the governing equation will disappear and the velocity takes on a constant value, $\bar{v}$. This is systems (6)-(8) become to the following form :

$$\bar{v} = Ra \int_0^1 (|\bar{\Theta}_r - \Theta_m|^b - |\bar{\Theta}_l - \Theta_m|^b) dz \quad (17)$$

$$\pm \bar{v} \frac{\partial \bar{\Theta}}{\partial z} = \frac{\partial^2 \bar{\Theta}}{\partial z^2} \quad \begin{matrix} +r \\ -l \end{matrix} \quad (18)$$

$$\bar{\Theta} = l \quad \text{at} \quad z = 0, \quad \bar{\Theta} = 0 \quad \text{at} \quad z = 1 \quad (19a,b)$$

The solution procedure of steady state problem is similar to that of the stability problem. The steady solution of equation (18) are

$$\bar{\Theta}_r = \frac{e^{\bar{v}z} - e^{\bar{v}}}{1 - e^{\bar{v}}}, \quad \bar{\Theta}_l = \frac{e^{-\bar{v}z} - e^{-\bar{v}}}{1 - e^{-\bar{v}}} \quad (20a,b)$$

After substituting (20) to (17) one can obtain the steady convection solution of density parameter Θ_m then we can obtain the steady convective solution of $\bar{v}$.

Results and Discussions

The method to derive the steady solutions of a thermosyphon system and their exchange of stability are described in above sections. Here we will show the results and discuss their characteristics. Fig. 2 shows that the onset critical Rayleigh number, Ra_c , will increase exponentially with increasing the magnitude of the temperature parameter of density inversion Θ_m . And when the Θ_m over the value 0.5, then $Ra_c \rightarrow \infty$, it means in that case the system will always keep in rest state.

Since the temperature parameter of density inversion, Θ_m , indicates the location point of density inverse, when Θ_m falls in the region between 0 and 1 ($T_U < T_m < T_D$), the cold water in the thermosyphon will occur the density inversion phenomenon. Initially, the system is static with the temperature distribution from the conduction solution, $1-z$. If $\Theta_m > 0.5$, the

maximum density appears at the lower half and the weight of fluid at the upper part is lighter than the lower. It is impossible to arise any buoyant phenomenon. Otherwise, if $\Theta_m < 0.5$, the heavier fluid concentrate to the upper in the rest state and induce the symmetric buoyancy force in each branch. Then, giving a perturbed v' which induce a disturbing viscous and a net disturbing buoyancy force. If the net buoyancy force less than the viscous force ($Ra < Ra_c$), the system will eliminate the perturbation and recover the static state. Opposite, $Ra > Ra_c$, it will onset the fluid flow.

Furthermore, the temperature distributions shows in Fig. 3. It is found that increasing the value of Θ_m make the temperature distribution more close to that of rest state.

Fig. 4 displays the steady flow velocity with decreasing the value of Θ_m . the result exhibits that the steady flow velocity increase with decreasing the value of Θ_m . this is because of smaller value of Θ_m induces larger value of steady flow velocity.

Conclusion

In this study, discussing the onset of motion and steady-state flows of cold water in a one dimension model parallel loops thermosyphon. It is show that if the temperature parameter of density inversion Θ_m between 0 and 1, the system will occur the density inversion phenomenon. And, increase the value of Θ_m will increase the magnitude of the onset critical Rayleigh number, Ra_c , and reduce the velocity of steady flow. Otherwise, this paper reveal that $\Theta_m > 0.5$, the system will always be kept in rest state.

Nomenclature

a	inner radius of tube	s_i	shape ratio $= a/h$
b	outer radius of tube	rsp	coefficient of density equation
g	gravity	T	temperature
h	high of loop	t	dimensionless time
P	modified Prandtl number	V	velocity
Pr	Prandtl number $= \alpha / \nu$	v	dimensionless velocity
Ra	Rayleigh number	Z	vertical coordinate
Ra_c	onset critical Rayleigh number	z	dimensionless vertical coordinate

Greek symbols

Θ	dimensionless temperature of fluid	ν	viscosity of fluid
ρ	density	σ	characteristic value
τ	time		

Subscripts

D	lower container	l	left branch
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m density extreme

U upper container

r right branch

Superscripts

$-$ steady state

$'$ perturbed state

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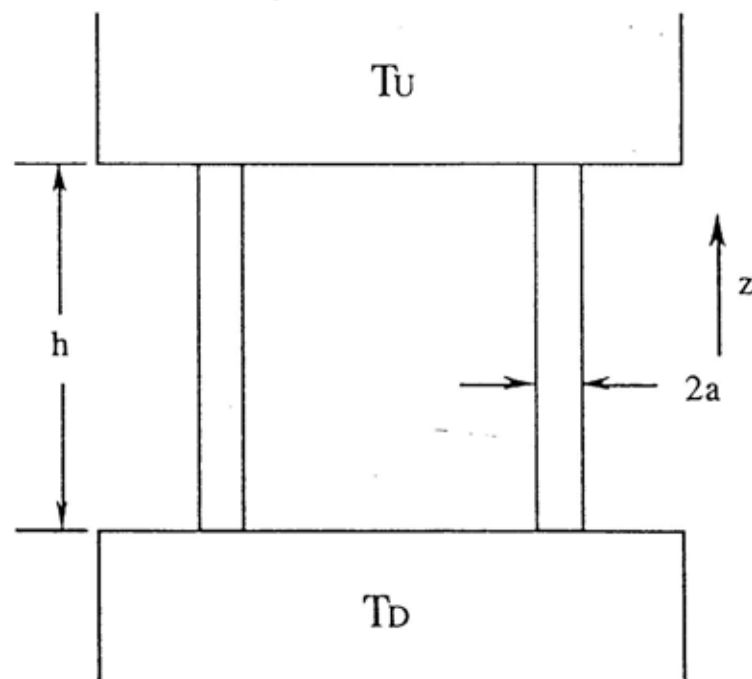
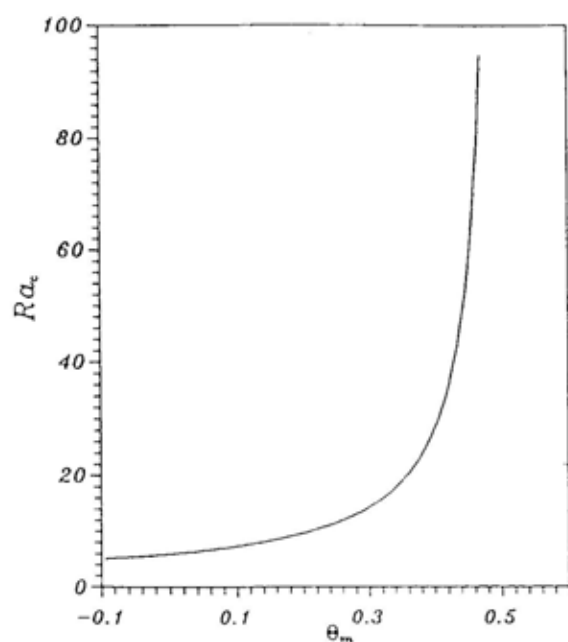
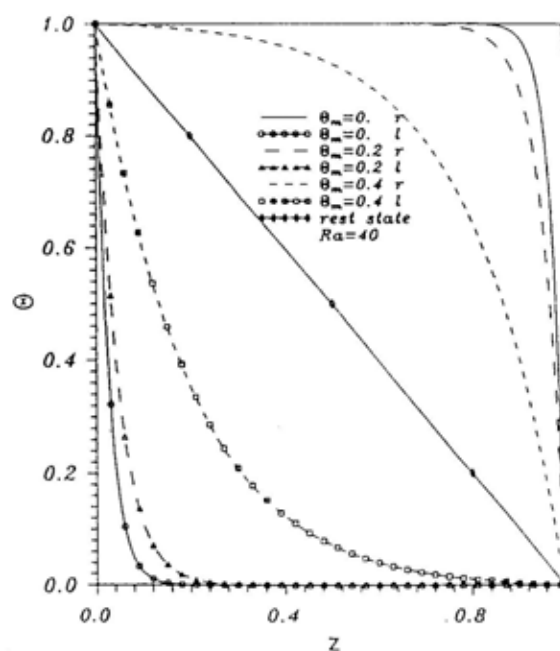
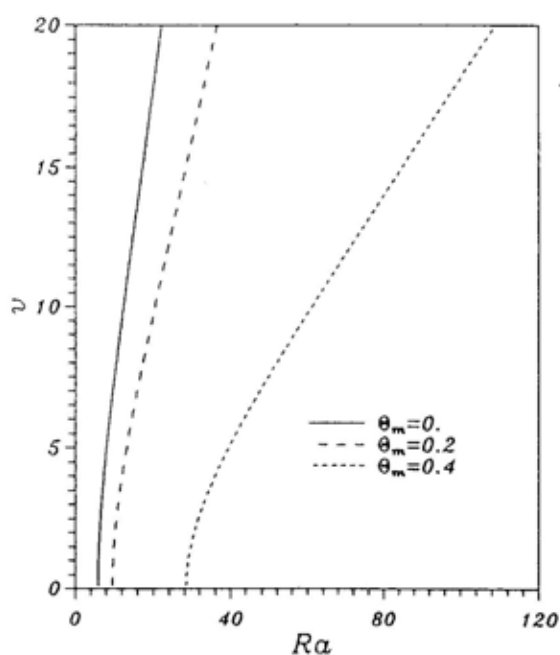


FIG. 1 A schematic description of the thermosyphon system

FIG. 2. Onset critical Rayleigh number v.s the Θ_m .FIG. 3. Steady temperature distribution with different values of Θ_m FIG. 4. Steady velocity, $\bar{v}$, v.s Ra with different values of Θ_m

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