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HYDRODYNAMIC LUBRICATION OF FINITE POROUS JOURNAL BEARINGS—USE OF THE BRINKMAN-EXTENDED DARCY MODEL

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Abstract—This paper applies the Brinkman-extended Darcy model (BEDM) to investigate the hydrodynamic lubrication of finite porous journal bearings. A finite difference technique employing the Preconditioned Conjugate Gradient method (PCGM) of iteration is used to solve the modified Reynolds equation. The results show that the influences of viscous shear stresses on the lubrication performance of porous journal-bearing systems are apparent and not negligible. Comparing with those of the Darcy model (DM), the effects of viscous shear stresses of the BEDM provide an increase in the load capacity, as well as a reduction in the friction parameter; and the quantitative effects of the viscous shear stresses on the static characteristics are more pronounced for the cases of large length-to-diameter ratios.

NOTATION

- b length of bearing
 C radial clearance, $r_1 - R$
 e eccentricity
 f coefficient of friction, F_j/W
 F_j friction force on the journal
 h film thickness
 h^* $h/C = 1 + \varepsilon \cos \theta$ (non-dimensional)
 H thickness of bearing matrix
 K permeability of bearing material
 p pressure in the oil film
 p^* $\frac{pC^2}{\mu U_2 R}$ (non-dimensional)
 r, θ, z cylindrical coordinates
 R radius of the journal
 r_1 inner radius of porous bearing
 r_0 outer radius of porous bearing
 u x-component of velocity in the film region
 W total load capacity of the bearing
 W_0 load capacity along the line of center
 W_0^* $\frac{W_0 C^2}{\mu U_2 b R^2}$ (non-dimensional)
 $W_{\pi/2}$ load capacity perpendicular to the line of center
 $W_{\pi/2}^*$ $\frac{W_{\pi/2} C^2}{\mu U_2 b R^2}$ (non-dimensional)
 x, y, z rectangular coordinates

Greek letters

- α $(\tilde{\mu}/\mu)^{1/2}$ (non-dimensional)
 β r_0/r_1 (non-dimensional)
 Δ load number, $\frac{W C^2}{\mu U_2 b R^2}$
 ε eccentricity ratio, e/C (non-dimensional)
 λ $\frac{b}{2r_1}$ (non-dimensional)
 μ oil viscosity
 $\tilde{\mu}$ viscosity of oil in the porous matrix
 σ $H/K^{1/2}$ (non-dimensional)
 σ_0 $C/K^{1/2}$ (non-dimensional)

- τ , shearing stress on the journal
 Φ permeability parameter, Kr_1/C^3 (non-dimensional)
 ψ attitude angle

1. INTRODUCTION

Self-lubricated porous journal bearings have been widely used in industry for a long time due to their self-contained "oil reservoir" and low costs. Lubricant initially in the bearings is supplemented by an external oil supply and when the journal picks up speed, the oil trapped in the pores of the bearing matrix is ejected and normally maintains a hydrodynamic film.

Traditionally, the analyses of hydrodynamic lubrication of porous journal bearings were based on the Darcy model (DM). In the DM Darcy's equation is used to depict oil motion through the porous matrix and a no-slip condition at the film/bearing interface is assumed. Morgan and Cameron [1] first gave a short-bearing solution. Cusano [2] analyzed the performance of finite porous journal using the Sommerfeld conditions and neglected the negative portion of the pressure distribution in calculating the load capacity. Murti [3] obtained an exact solution for the Laplace equation in cylindrical coordinates of finite porous journal bearings with arbitrary wall thickness. Murti [4] assessed the lubrication performance of finite porous bearings following the Rhodes-Rouleau approach [5] under the half-film condition.

In 1966, Ishizawa and Hori [6] analyzed the axisymmetric flow through a porous wall into a narrow and uniform gap. They verified theoretically the existence of a slip of fluid on the porous wall surface and clarified quantitatively the limit of applicability of the no-slip condition. For the flow through a porous wall into a narrow gap, it was necessary to take account of the phenomena of slip. Later, Beavers and Joseph [7] and Beavers *et al.* [8] showed experimentally that the effect of the viscous shear stress in the fluid layer might penetrate into the permeable layer for flow in superposed layers. Beavers and Joseph retained the Darcy equation and proposed a slip velocity boundary condition at the fluid/porous interface. They assumed the slip velocity for the free fluid to be:

- (i) proportional to the shear rate at the permeable boundary;
- (ii) related to the exterior flow by the appropriate boundary condition;

and defined a dimensionless slip coefficient, α , which depends upon the variation of the porosity. According to their results, the value of $\alpha = 0.1$ is used for aloxite porous specimens, and the value of $\alpha = 0.78 \sim 4.01$ is used for foametal porous specimens. Murti [9] thus applied the slip-velocity boundary condition to examine the influence of slip effect on the pressure distribution of long porous bearing. Murti [10], and Prakash and Vij [11] also investigated the characteristics of narrow porous journal bearings using the slip-flow model. Rouleau and Steiner [12] derived a modified Reynolds equation for finite porous bearings considering slip at the film/bearing interface. Kaneko [13], and Kaneko and Doi [14], respectively, examined the static and dynamic characteristics of porous journal bearings with anisotropic permeability and with non-uniform permeability. Bujurke and Naduvanamani [15] also applied the slip-flow model to investigate the performance of narrow porous journal bearings lubricated by couple stress fluid.

Recently, many works by Neale and Nader [16], Nield [17], and Rudraiah [18] have applied the Brinkman-extended Darcy equation with constant porosity to investigate the flow in superposed fluid and porous medium. Since the Brinkman-extended Darcy equation includes a viscous shear term matching the shearing stress across the fluid/porous matrix interface, the artificial slip-flow model is not needed. The analysis by Neale and Nader [16] proposed that the Brinkman-extended Darcy model (BEDM) with constant porosity is identical to the Beavers and Joseph slip-flow model if the parameter α is defined as the following:

$$\alpha = \left(\frac{\tilde{\mu}}{\mu} \right)^{1/2}, \quad (1)$$

where $\tilde{\mu}$ denotes the viscosity of fluid in the permeable matrix which is assumed to be different in value from μ , the viscosity of the fluid layer. Lin and Hwang [19, 20] first applied

the BEDM to investigate the hydrodynamic lubrication of short and long porous journal bearings. They found that, comparing with those of the DM, the viscous shear effects increased the load capacity and decreased the friction parameter of short and long porous journal bearings, but we have no idea, on the basis of the BEDM, how the viscous shear stresses affect the static characteristics of finite porous journal bearings. Thus, an examination of the influences of the viscous shear stresses is needed.

This paper applies the BEDM to investigate the viscous shear effects on the lubrication performance of finite porous journal bearings. The pressure in the film is numerically obtained by employing the PCGM (Preconditioned Conjugate Gradient Method) of iteration. Other bearing characteristics, e.g. the load capacity, friction parameter and attitude angle are also evaluated. A comparison between the results of the BEDM and the DM is made to predict the viscous shear effects on the static characteristics of finite porous journal bearings, and the results for the limiting case (Φ approaches zero) of the BEDM giving a good agreement with those of a solid bearing, support the present predictions.

2. ANALYSIS

2.1. Modified Reynolds equation

Figure 1 shows the physical configuration of the porous journal bearing used in this analysis. The porous bearing is press-fitted in a housing and supports a non-porous journal through an oil film. Assume that: (i) the film is thin, (ii) there is no variation of pressure across the film, (iii) there is no external force acting on the film, (iv) fluid inertia is small compared to the viscous shear stress, (v) the lubricant is an incompressible Newtonian fluid and has constant properties, and (vi) the flow is in steady state. Also, assume that the porous bearing material is homogeneous and isotropic, the flow of lubricant in the bearing is laminar, and the wall thickness of the porous bearing is thin, i.e. the variation of pressure across the porous medium is negligible. According to Lin and Hwang [19], the modified Reynolds equation is:

$$\frac{\partial}{\partial x} \left(C^3 A^* \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(C^3 A^* \frac{\partial p}{\partial z} \right) = 6\mu U_2 (1 + B^*) \frac{\partial h}{\partial x}, \quad (2)$$

where A^* and B^* are given in the Appendix.

Introducing the dimensionless variables and parameters:

$$\theta = \frac{x}{r_1}, \quad (3a)$$

$$z^* = \frac{z}{b}, \quad (3b)$$

$$p^* = p \frac{C^2}{\mu R U_2}, \quad (3c)$$

$$\lambda = \frac{b}{2r_1}, \quad (3d)$$

$$\beta = \frac{r_0}{r_1}, \quad (3e)$$

and:

$$\Phi = \frac{K r_1}{C^3} \quad (3f)$$

into the modified Reynolds equation (2) yields:

$$\frac{\partial}{\partial \theta} \left(A^* \frac{\partial p^*}{\partial \theta} \right) + \frac{1}{4\lambda^2} \frac{\partial}{\partial z^*} \left(A^* \frac{\partial p^*}{\partial z^*} \right) = \frac{6(1 + B^*)}{1 - C/r_1} \frac{\partial h^*}{\partial \theta}. \quad (4)$$

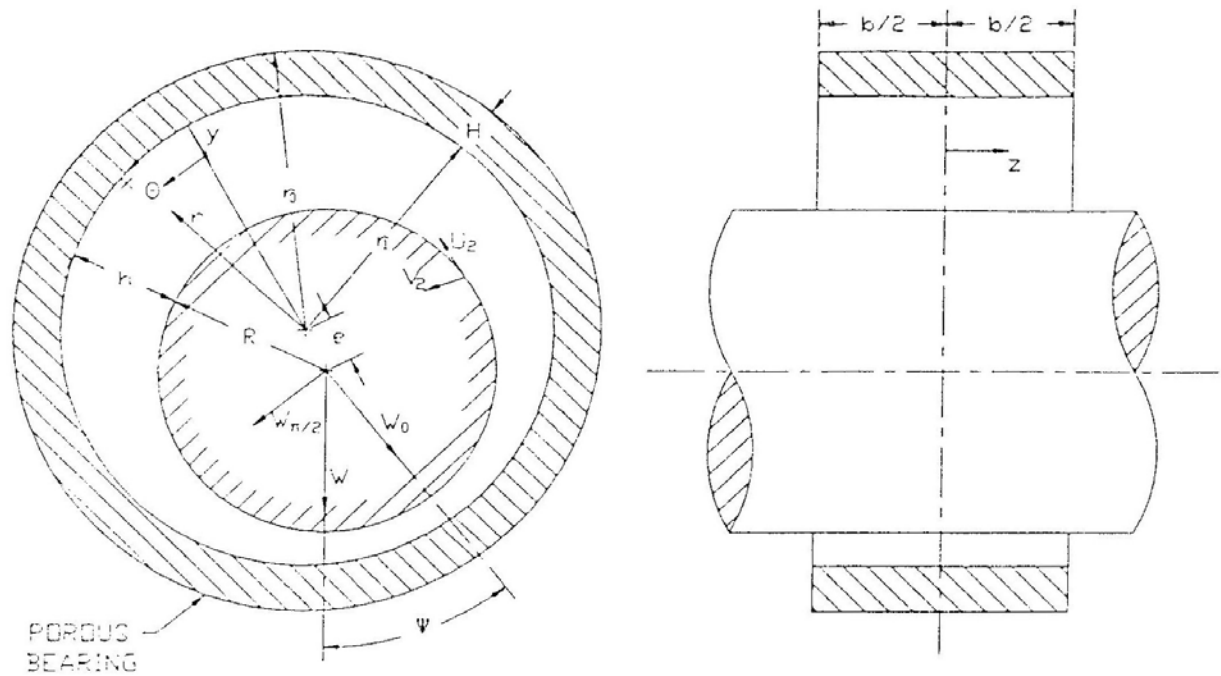


FIG. 1. Porous journal bearing configuration.

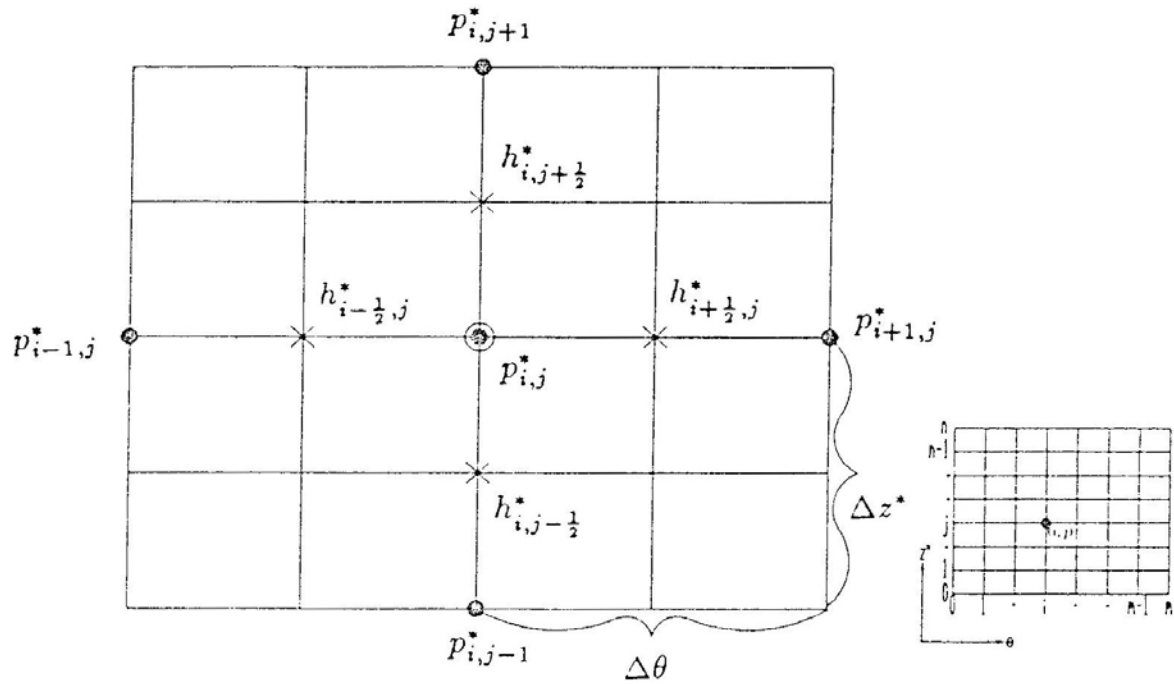


FIG. 2. Grid points notation.

2.2. Boundary conditions

In a previous work studying the static characteristics of finite porous journal bearings, the π -film condition was used by Murti [4]. For the convenience of comparison we assume that the boundary conditions for the lubricant in the film region are:

$$p^* = 0 \quad \text{at } \theta = 0 \quad \text{and } \pi, \quad (5)$$

and:

$$p^* = 0 \quad \text{at } z^* = -\frac{1}{2} \quad \text{and } \frac{1}{2}. \quad (6)$$

2.3. Numerical formulation

The modified Reynolds equation is solved numerically using a finite difference scheme. The film domain under consideration is divided by grid spacing. Referring to Fig. 2, we

have, in finite increment format, the three terms in Eqn (4) as:

$$\frac{\partial}{\partial \theta} \left(A^* \frac{\partial p^*}{\partial \theta} \right) = \frac{A_{i+\frac{1}{2},j}^* \frac{p_{i+1,j}^* - p_{i,j}^*}{\Delta \theta} - A_{i-\frac{1}{2},j}^* \frac{p_{i,j}^* - p_{i-1,j}^*}{\Delta \theta}}{\Delta \theta}, \quad (7)$$

$$\frac{\partial}{\partial z^*} \left(A^* \frac{\partial p^*}{\partial z^*} \right) = \frac{A_{i,j+\frac{1}{2}}^* \frac{p_{i,j+1}^* - p_{i,j}^*}{\Delta z^*} - A_{i,j-\frac{1}{2}}^* \frac{p_{i,j}^* - p_{i,j-1}^*}{\Delta z^*}}{\Delta z^*}, \quad (8)$$

$$(1 + B^*) \frac{\partial h^*}{\partial \theta} = (1 + B_{i,j}^*) \frac{h_{i+\frac{1}{2},j}^* - h_{i-\frac{1}{2},j}^*}{\Delta \theta}. \quad (9)$$

From the above, we can solve $p_{i,j}^*$ as:

$$p_{i,j}^* = a_0 + a_1 p_{i+1,j}^* + a_2 p_{i-1,j}^* + a_3 p_{i,j+1}^* + a_4 p_{i,j-1}^*, \quad (10)$$

where:

$$a_0 = \frac{-24\lambda^2 r^* \Delta z^* (1 + B_{i,j}^*) (h_{i+\frac{1}{2},j}^* - h_{i-\frac{1}{2},j}^*)}{(1 - C/r_1) \delta}, \quad (11a)$$

$$a_1 = \frac{4\lambda^2 r^{*2} A_{i+\frac{1}{2},j}^*}{\delta}, \quad (11b)$$

$$a_2 = \frac{4\lambda^2 r^{*2} A_{i-\frac{1}{2},j}^*}{\delta}, \quad (11c)$$

$$a_3 = \frac{A_{i,j+\frac{1}{2}}^*}{\delta}, \quad (11d)$$

$$a_4 = \frac{A_{i,j-\frac{1}{2}}^*}{\delta}, \quad (11e)$$

$$r^* = \frac{\Delta z^*}{\Delta \theta}, \quad (11f)$$

and:

$$\delta = 4\lambda^2 r^{*2} (A_{i+\frac{1}{2},j}^* + A_{i-\frac{1}{2},j}^*) + (A_{i,j+\frac{1}{2}}^* + A_{i,j-\frac{1}{2}}^*). \quad (11g)$$

For $m \times n$ points in the mesh there will be $m \times n$ simultaneous equations which can be solved by the PCGM of iteration [21]. The mesh has 30 intervals in the θ direction, and 10 intervals across the bearing width b . The tolerance of the convergence criterion is 0.0001.

3. BEARING CHARACTERISTICS

With the dimensionless film pressure known, all the bearing characteristics can be determined by numerical integration using Gaussian quadrature.

3.1. Load capacity

The load capacity is obtained by integrating the film pressure. The integration in the circumferential direction is taken from $\theta = 0$ to $\theta = \pi$ under the half-film condition. The load component along the line of center W_0 is given by:

$$W_0 = -2R \int_{z=0}^{z=b/2} \int_{\theta=0}^{\theta=\pi} p \cos \theta \, d\theta \, dz, \quad (12)$$

and the load component perpendicular to the line of center $W_{\pi/2}$ is given by:

$$W_{\pi/2} = 2R \int_{z=0}^{z=b/2} \int_{\theta=0}^{\theta=\pi} p \sin \theta \, d\theta \, dz. \quad (13)$$

Expressed in terms of dimensionless quantities, the load components are given by:

$$W_0^* = \frac{W_0}{\mu U_2 b} \frac{C^2}{R^2} = -2 \int_{z^*=0}^{z^*=1/2} \int_{\theta=0}^{\theta=\pi} p^* \cos \theta \, d\theta dz^*, \quad (14)$$

and

$$W_{\pi/2}^* = \frac{W_{\pi/2}}{\mu U_2 b} \frac{C^2}{R^2} = 2 \int_{z^*=0}^{z^*=1/2} \int_0^\pi p^* \sin \theta \, d\theta dz^*. \quad (15)$$

Let the load number Δ be:

$$\Delta = \frac{W}{\mu U_2 b} \frac{C^2}{R^2}. \quad (16)$$

Since the load capacity is

$$W = (W_0^2 + W_{\pi/2}^2)^{1/2}, \quad (17)$$

the load number is given by:

$$\Delta = (W_0^{*2} + W_{\pi/2}^{*2})^{1/2}. \quad (18)$$

3.2. Attitude angle

The attitude angle ψ can be obtained from Eqns (14) and (15) as:

$$\psi = \tan^{-1} \left(\frac{W_{\pi/2}^*}{W_0^*} \right). \quad (19)$$

3.3. Coefficient of friction

The shear stress acting on the journal is:

$$\tau_s = \mu \frac{\partial u}{\partial y} \bigg|_{y=h}. \quad (20)$$

The friction force acting on the journal is:

$$F_j = \int_0^{2\pi} \tau_s b R \, d\theta = b R \int_0^{2\pi} \mu \frac{\partial u}{\partial y} \bigg|_{y=h} \, d\theta \quad (21)$$

The coefficient of friction f is obtained by F_j/W , and the friction parameter fR/C is:

$$f \frac{R}{C} = \frac{2\pi}{\Delta \left[\left(1 + \frac{1}{\alpha \sigma_0} \frac{1 - \exp\left(-\frac{2\sigma}{\alpha}\right)}{1 + \exp\left(-\frac{2\sigma}{\alpha}\right)} \right)^2 - \varepsilon^2 \right]^{1/2}} + \left(1 - \frac{C}{r_1} \right) \int_0^{2\pi} \frac{\left[1 + \exp\left(\frac{-2\sigma}{\alpha}\right) \right] h^{*2} + \left[1 - \exp\left(\frac{-2\sigma}{\alpha}\right) \right] \frac{2}{\alpha \sigma_0} h^* + \left[1 - \exp\left(\frac{-\sigma}{\alpha}\right) \right]^2 \frac{2}{\sigma_0^2}}{2 \left\{ \left[1 + \exp\left(\frac{-2\sigma}{\alpha}\right) \right] h^* + \left[1 - \exp\left(\frac{-2\sigma}{\alpha}\right) \right] \frac{1}{\alpha} \right\} \Delta} \left(\frac{dp^*}{d\theta} \right) d\theta. \quad (22)$$

4. RESULTS AND DISCUSSION

This paper applies the BEDM to predict the viscous shear effects on the lubrication performance of finite porous journal bearings. The pressure in the film is numerically obtained by the PCGM of iteration. Then the film pressure is utilized to evaluate the load number, the friction parameter and the attitude angle. The chosen parameters in the BEDM and the DM are $\beta = 1.2$, $C/r_1 = 0.001$.

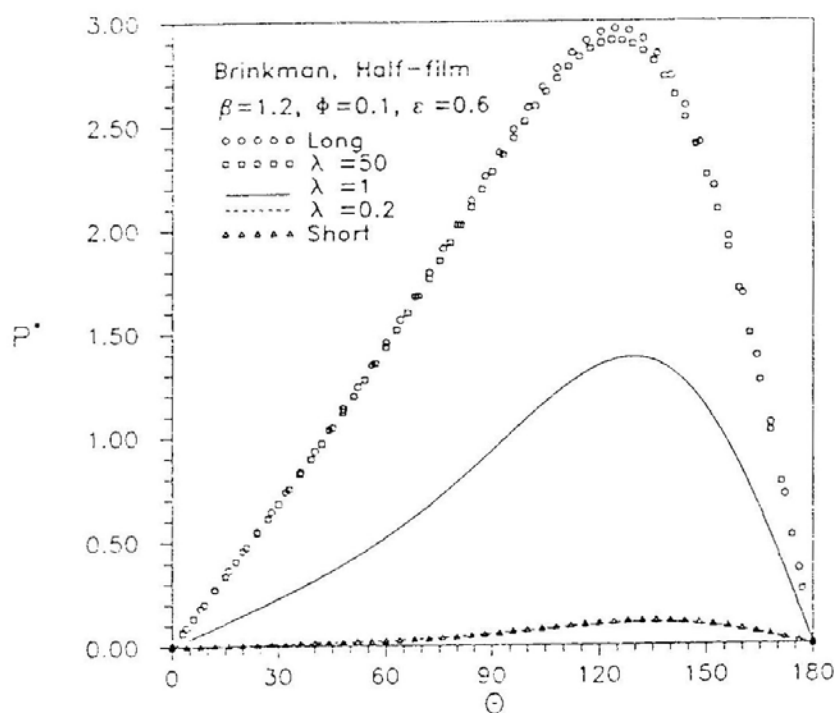
The value of α depends on the variation of the porosity, as we have shown. The existence of α provides the viscous shear effects of the BEDM on the lubrication performance of finite porous journal bearings. The effect of α on the dimensionless film pressure and the friction parameter is presented in Tables 1 and 2. Table 1 shows that the larger the value of α is, the

TABLE 1. FILM PRESSURE UNDER HALF-FILM CONDITION,
 $\beta = 1.1, \lambda = 1.0, \varepsilon = 0.2, \Phi = 0.01$

	$\alpha = 0.78$	$\alpha = 1.00$	$\alpha = 1.45$	$\alpha = 4.01$
θ	P^*	P^*	P^*	P^*
000.0	0.0000	0.0000	0.0000	0.0000
020.0	0.0980	0.0982	0.0985	0.0989
040.0	0.1957	0.1962	0.1967	0.1975
060.0	0.2904	0.2911	0.2918	0.2930
080.0	0.3740	0.3749	0.3759	0.3775
100.0	0.4305	0.4316	0.4329	0.4347
120.0	0.4355	0.4367	0.4380	0.4400
140.0	0.3645	0.3656	0.3667	0.3684
160.0	0.2104	0.2111	0.2117	0.2127
180.0	0.0000	0.0000	0.0000	0.0000

TABLE 2. FRICTION PARAMETER UNDER HALF-FILM CONDITION, $\beta = 1.3$,
 $\lambda = 1.0, \varepsilon = 0.6$

	$\alpha = 0.78$	$\alpha = 1.00$	$\alpha = 1.45$	$\alpha = 4.01$
Φ	$f(R/C)$	$f(R/C)$	$f(R/C)$	$f(R/C)$
0.0001	7.4594	7.4590	7.4585	7.4571
0.0003	7.4787	7.4781	7.4769	7.4747
0.0010	7.5337	7.5325	7.5303	7.5264
0.0032	7.6915	7.6894	7.6861	7.6795
0.0100	8.1255	8.1232	8.1186	8.1089
0.0316	9.2073	9.2076	9.2040	9.1931
0.1000	11.6723	11.6859	11.6926	11.6901
0.3162	17.3831	17.4567	17.5169	17.5731
1.0000	32.1001	32.4797	32.8270	33.2290



higher the dimensionless film pressure is, but it is found that the effect of α on p^* is small and negligible. Table 2 displays the effect of the variations of α on the friction parameter. Also, we see that the effect of variation of α on the friction parameter is small and can be neglected. Without loss of generality, we adopt the value of $\alpha = 1$, which is suggested by Neale and Nader [16], in the following calculation.

4.2. Film pressure

Figure 3 shows the dimensionless pressure distribution at different λ (length-to-diameter ratio). The results for the short- and long-bearing approximations were obtained by Lin and Hwang [19, 20]. The curves for $\lambda = 0.2$, $\lambda = 1$ and $\lambda = 50$ are numerically obtained from the finite-bearing Eqn (4). It shows that the dimensionless film pressure distribution for a finite bearing for $\lambda = 0.2$ is close to that of a short-bearing approximation, and, for $\lambda > 50$, to that of a long-bearing approximation.

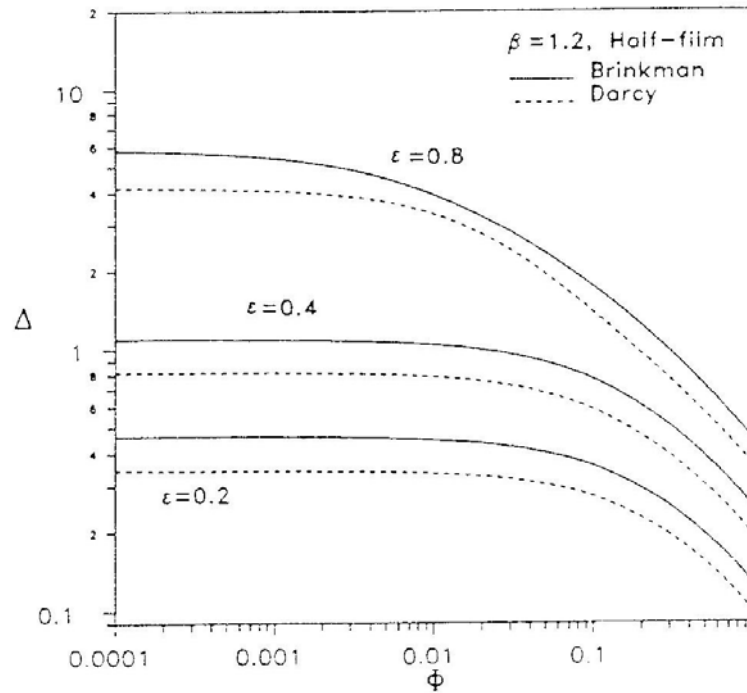
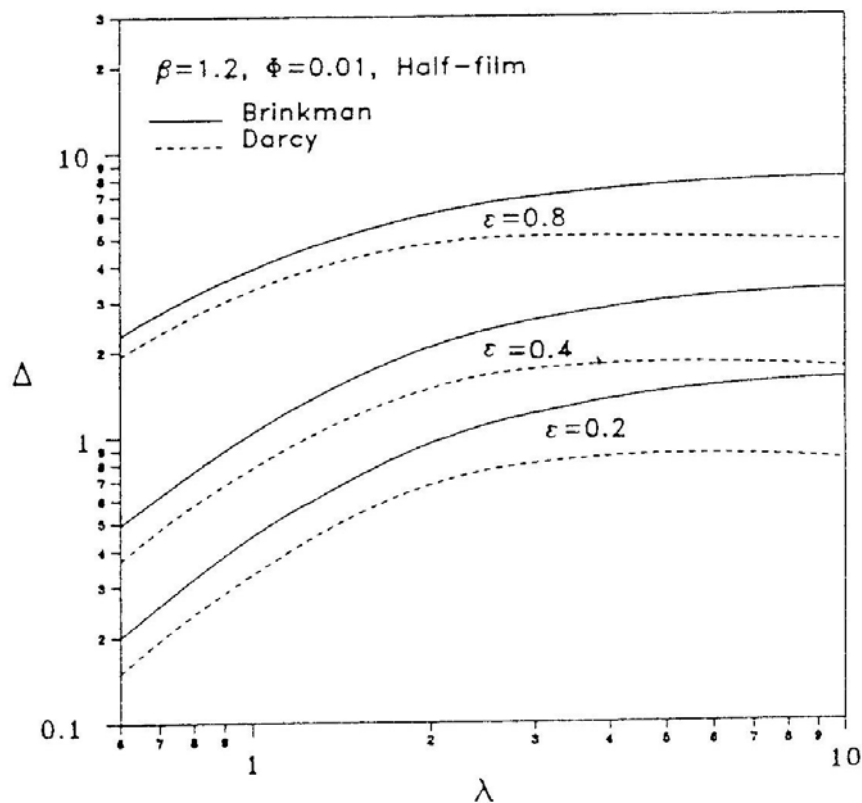


FIG. 4. The load number Δ at different ε resulting from the BEDM and the DM for $\lambda = 1$.



4.3. Load capacity

Figure 4 shows the load numbers Δ vs permeability parameter Φ at different eccentricity ratio ε for $\lambda = 1$. The results obtained from the DM [4] are plotted in dashed curves. It is seen that the BEDM yields a higher load number. In the DM the Darcy's equation neglects the viscous shear term. Since the Brinkman-extended Darcy equation includes the viscous shear term matching the shearing stress across the film/bearing interface, the viscous shear effects of the BEDM enhance the load number. Figure 5 displays the load numbers vs λ at different ε . It is found that the viscous shear effects of the BEDM always increase the load number. For fixed ε the larger the value of λ is, the more the viscous shear effects increase the load number. Figure 6 shows the load numbers vs λ at different Φ . For fixed Φ it is also

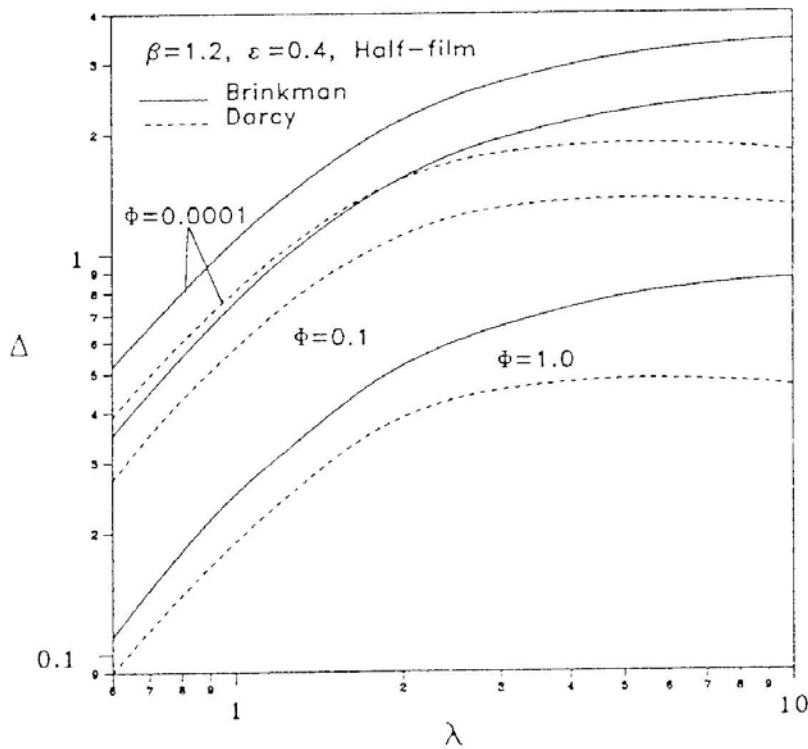


FIG. 6. Comparison of the load number Δ vs λ at different Φ resulting from the BEDM and the DM.

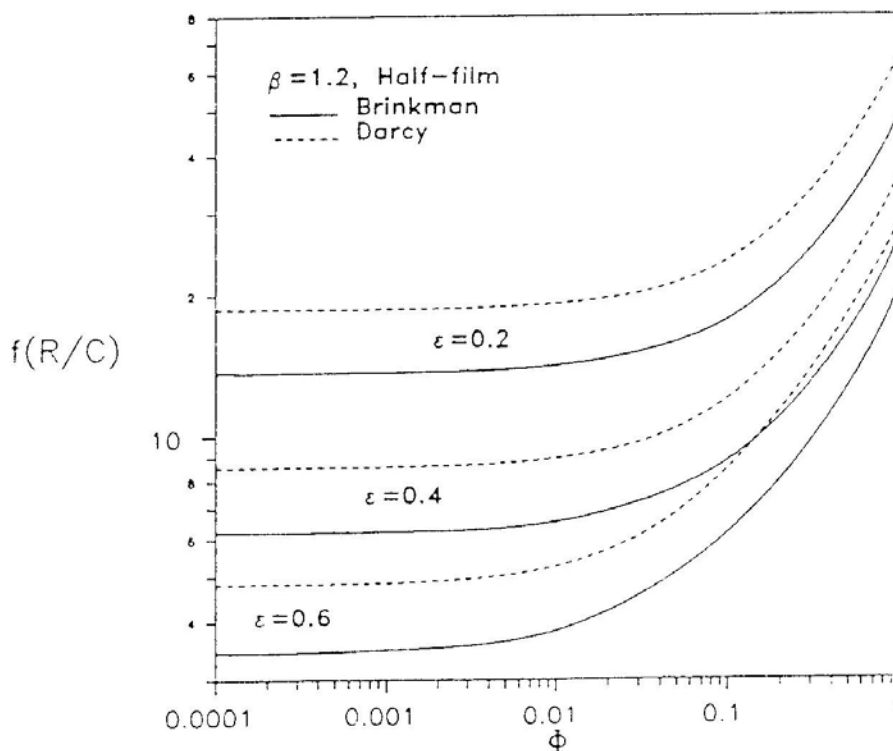


FIG. 7. The friction parameter $f(R/C)$ at different ε resulting from the BEDM and the DM for $\lambda = 1$.

found that the larger the value of λ is, the more the viscous shear effects enhance the load number.

Overall, it is found that the viscous shear effects of the BEDM provide an increase in the load capacity of porous journal bearings. In addition, the present results confirm the conclusions by Lin and Hwang [19, 20], respectively, for the short- and long-bearing approximations.

4.4. Friction parameter

Figure 7 shows the comparison of the friction parameter fR/C between the BEDM and the DM for $\lambda = 1$. It is found that the viscous shear effects of the BEDM decrease the

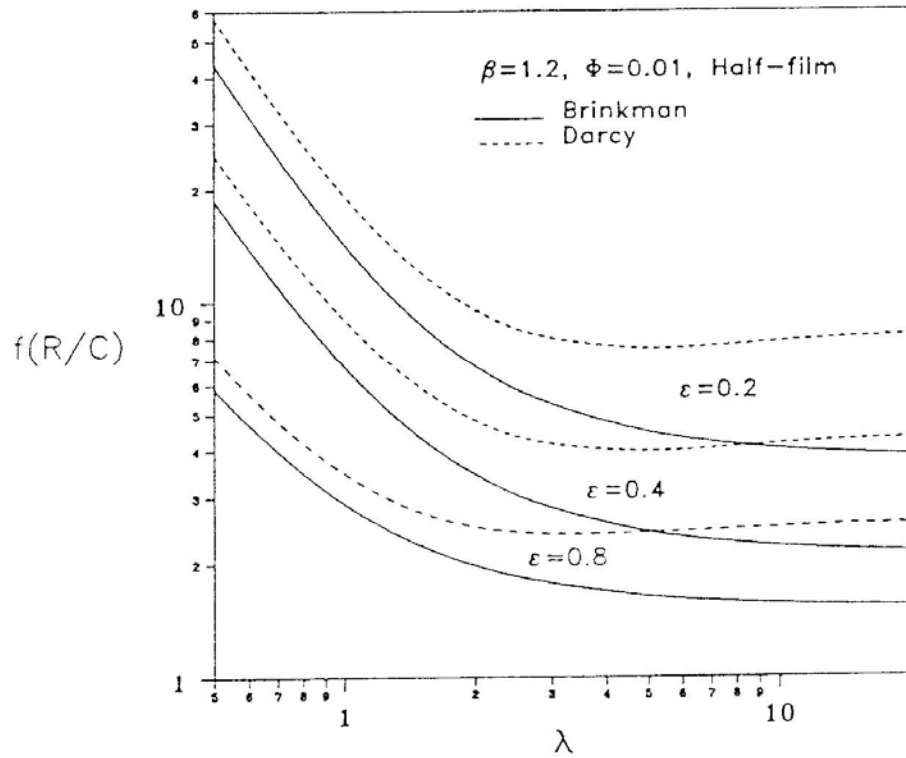


FIG. 8. Comparison of the friction parameter $f(R/C)$ vs λ at different ϵ resulting from the BEDM and the DM.

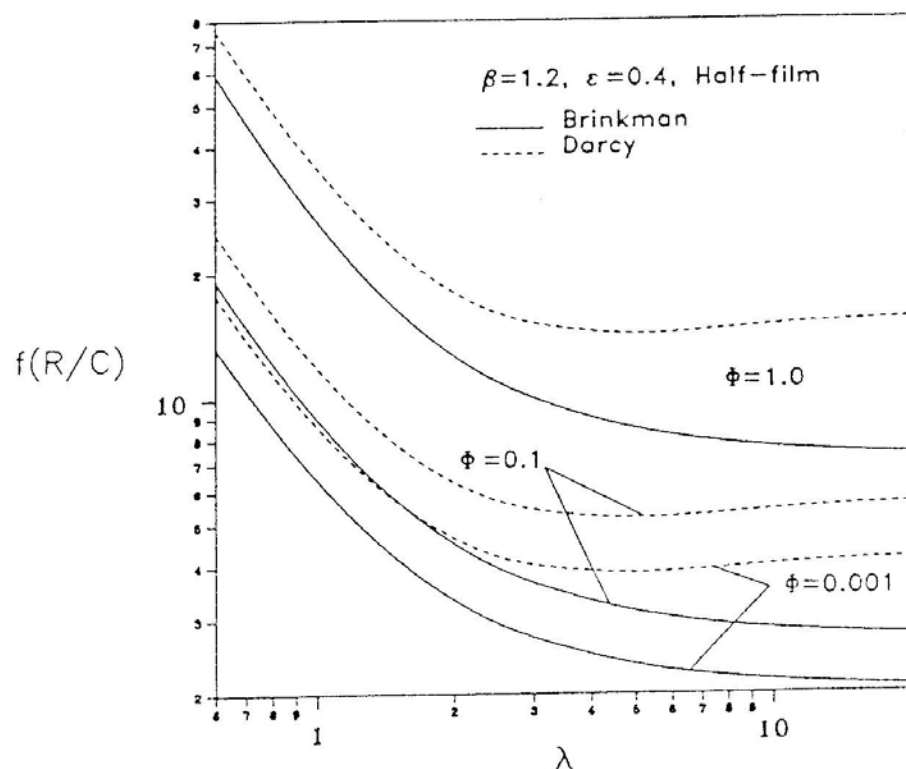


FIG. 9. Comparison of the friction parameter $f(R/C)$ vs λ at different Φ resulting from the BEDM and the DM.

friction parameter, and, when the value of Φ decreases, the decrement is enlarged. Figure 8 displays the friction parameter vs λ for $\Phi = 0.01$. It shows that the BEDM results in a lower friction parameter compared to that of the DM. For fixed ε the larger the value of λ is, the more the viscous shear effects reduce the friction parameter. Figure 9 shows the friction parameters vs λ at $\varepsilon = 0.4$. It also shows that the BEDM results in a lower friction parameter, and for fixed Φ the larger the value of λ is, the more influence the viscous shear effects have on the friction parameter.

Totally, it is found that the BEDM predicts a decreased friction parameter compared to that of the DM, and the longer the bearing is, the more the viscous shear effects reduce the friction parameter. For the short-bearing ($\lambda \ll 1$) and long-bearing ($\lambda \gg 1$) limits the results confirm the conclusions by Lin and Hwang [19, 20], respectively, for the short- and long-bearing approximations.

4.5. Attitude angle

Figure 10 displays the attitude angle (degree) vs Φ for $\lambda = 1$. It shows that for low Φ the BEDM gives a lower attitude angle than that of the DM, and the higher the value of ε is, the more the amount of decrement in ψ becomes. But, for high Φ , the BEDM results in a visible increase in value of ψ . Figure 11 shows the value of ψ (degree) vs λ for $\Phi = 0.01$. It is found that, for small λ , the BEDM predicts a lower attitude angle compared to that of the DM, but for $\lambda \gg 1$, the BEDM yields a higher attitude angle especially at high ε ($\varepsilon = 0.8$).

4.6. Compared to that of a solid bearing

As the value of permeability parameter Φ approaches zero, the characteristics of the porous bearing approach that of a solid bearing. Thus, irrespective of the model used, the data for very small Φ should be about the same. Figure 12 shows the comparison of the static characteristics resulting from the BEDM, the DM and the solid bearing for $\lambda = 1$. The data of the solid bearing are obtained by Pinkus and Sernlicht [22]. The results of the BEDM and the DM are obtained as Φ approaches zero. It is found that the load number Δ , the friction parameter fR/C and the attitude angle ψ resulting from the BEDM agree well with those of the solid bearing. The good agreement with that of the solid bearing, thus provides a support to the present predictions.

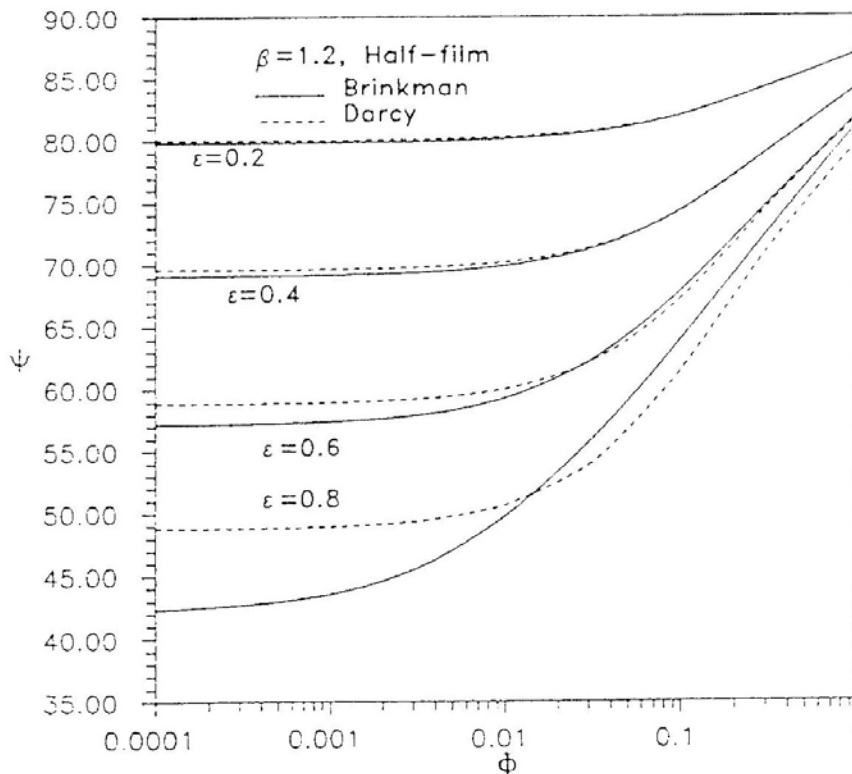


FIG. 10. The attitude angle ψ (degree) at different ε resulting from the BEDM and the DM for

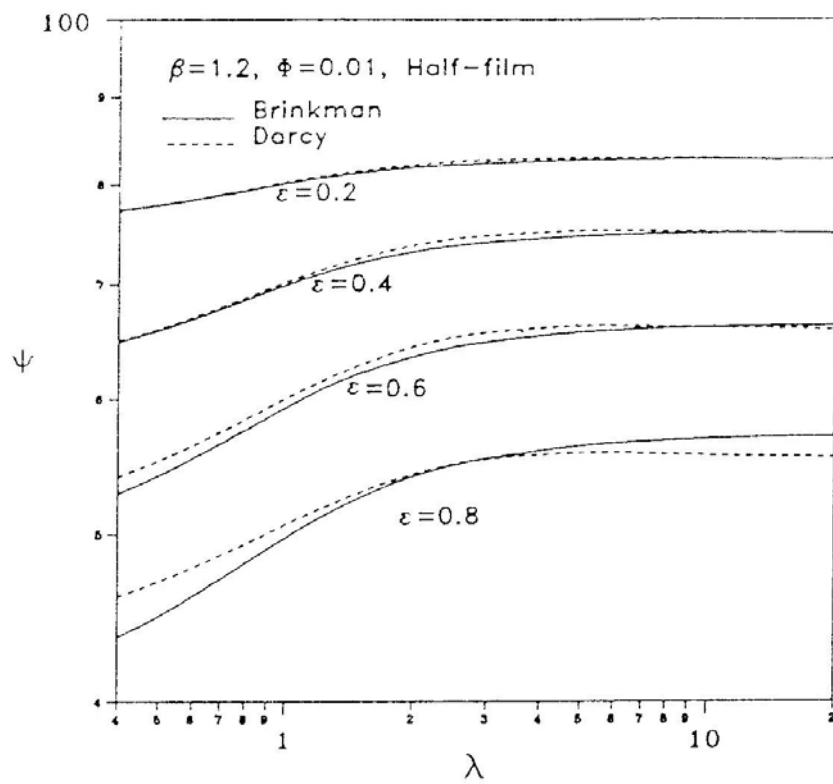


FIG. 11. Comparison of the attitude angle ψ (degree) vs λ at different ε resulting from the BEDM and the DM.

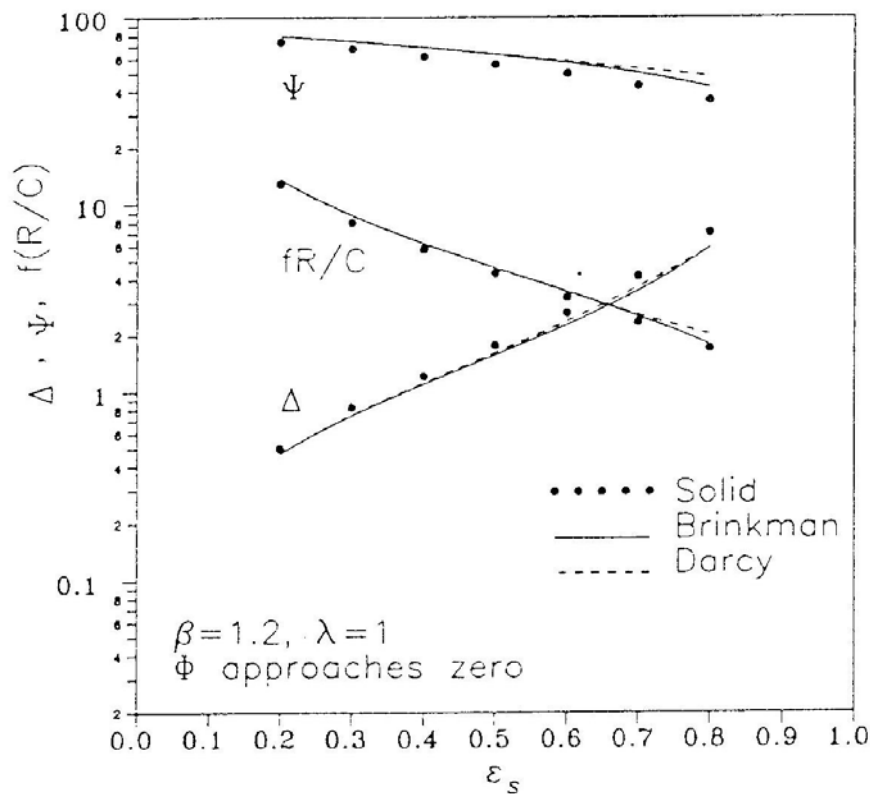


FIG. 12. Comparison of the static characteristics resulting from the BEDM, the DM and the solid bearing.

4.7. Previous work about slip effect

Based on the DM with slip effect at the film/bearing interface, Rouleau and Steiner [12] have investigated the hydrodynamic lubrication of finite porous journal bearings. According to their conclusion, the effect of slip at the interface between the lubricant film and the porous bearing is negligible for the usual range of bearing parameters, but from above we find that the effects of viscous shear stresses on the porous journal-bearing system cannot be

On the basis of the BEDM, this paper investigates the viscous shear effects on the static characteristics of finite porous journal bearings under half-film conditions. The film pressure is numerically solved by using the PCGM and applied to predict the static characteristics. To support the theoretical predictions, the results are compared to those of a solid bearing. As the value of permeability parameter Φ approaches zero, the good agreement with that of the solid bearing supports the present predictions. Comparing with that of the DM under half-film condition [4], the conclusion can be drawn as follows.

- (i) For different porous media, the effect of variation of α on the bearing characteristics is small and negligible.
- (ii) The viscous shear effects enhance the load capacity as well as reduce the friction parameter. For the cases of short-bearing and long-bearing limits the results confirm, respectively, the conclusions of short-bearing and long-bearing approximations by Lin and Hwang [19, 20].
- (iii) The viscous shear effects decrease the attitude angle for small λ , but increase the attitude angle for large λ .
- (iv) On the whole, the effects of viscous shear stresses of the BEDM on the lubrication performance of porous journal-bearing systems are apparent and not negligible. However, the effects on the dynamic lubrication problem remain to be investigated.

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APPENDIX

The expressions of A^* and B^*

$$A^* = h^{*3} + A_1^* h^{*2} + A_2^* h^* + A_3^* - \frac{A_6^*}{A_4^* h^* + A_5^*} \quad (\text{A1})$$

$$A_1^* = \frac{1 - \exp\left(-\frac{2\sigma}{\alpha}\right)}{1 + \exp\left(-\frac{2\sigma}{\alpha}\right)} \frac{3}{\alpha\sigma_0} \quad (\text{A2})$$

$$A_2^* = \frac{12\alpha^2 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right] \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right)\right]^2 - 3 \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^2}{\left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^2} \frac{1}{\alpha^2 \sigma_0^2} \quad (\text{A3})$$

$$A_3^* = \frac{3 \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^3 - 12\alpha^2 \left[1 - \exp\left(-\frac{4\sigma}{\alpha}\right)\right] \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right)\right]^2}{\alpha^3 \sigma_0^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^3} + \frac{12\sigma\alpha^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^3 - 12\alpha^4 \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right)\right] \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^2}{\alpha^3 \sigma_0^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^3} \quad (\text{A4})$$

$$A_4^* = \alpha^3 \sigma_0^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^3 \alpha\sigma_0 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right] \quad (\text{A5})$$

$$A_5^* = \alpha^3 \sigma_0^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^3 \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right)\right] \quad (\text{A6})$$

$$A_6^* = 24\alpha^4 \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right)\right]^2 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^3 - 12\alpha \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^2 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^2 - 12\alpha^2 \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right)\right]^2 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right] \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^2 + 3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^4 \quad (\text{A7})$$

$$B^* = \frac{\left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right)\right]^2 - 2\alpha^2 \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right)\right]^2 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right]}{\left\{ \alpha\sigma_0 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right)\right] h^* + \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right)\right] \right\}^2} \quad (\text{A8})$$

$$\sigma = \frac{H}{K^{1/2}} \quad (\text{A9})$$

$$\sigma_0 = \frac{C}{K^{1/2}}, \quad (\text{A10})$$