

Effects of roughness orientations on thin film lubrication of a magnetic recording system

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Abstract. In this paper, the relations expressing the effects of roughness orientations and molecular mean free path on the roughness-induced flow factors are derived. Such consideration of the effects of roughness orientation on bearing performance is necessary when rotary actuators are used in magnetic recording systems. Using the perturbation approach and the coordinate transformation, the flow factors are derived and expressed as functions of the roughness orientations, the Peklenik numbers, the standard deviations of roughness heights of each surfaces, the inverse Knudsen number and the film thickness ratio of the lubrication film, where h/σ is the ratio of average film thickness to standard deviation of the composite roughness. Two orientation coefficients are proposed to reveal the combined effects of Peklenik number and orientation angle and the change of flow factors is analysed as the orientation coefficients vary. Finally, the static performance of a magnetic recording head flying above a rotating disc under the ultra-thin-spacing condition is analysed with roughness and rarefaction effects taken into account. The effects of surface characteristics of the moving surface on load capacity are more significant than those of the stationary surface.

1. Introduction

The reduction in spacing between the magnetic head and recording medium is of great importance in increasing the recording density in magnetic systems. It is more appropriate to reduce the slider bearing area, by reducing the width of slider rails, than to increase the spring load on the slider or to decrease the disc velocity. Given such low spacing (less than $0.1\ \mu\text{m}$) and the small area of the slider, it becomes important to consider the effects of rarefied gas and roughness on the magnetic recording systems.

In thin-film gas lubrication, the modified Reynolds equations based on the velocity boundary of the first-order slip flow [2], second-order slip flow [3] or of slip flow order 1.5 [4] for small Knudsen number (K_n), defined as the ratio of the molecular mean free path (λ) to spacing (h), have been widely used as lubrication equations for conventional flying head sliders. For arbitrary Knudsen number conditions, analyses based on the Boltzmann equation taking molecular behaviour into account have also been investigated. Furthermore, the generalized lubrication equation applicable to the ultra-thin spacing of less than $0.1\ \mu\text{m}$ was derived [5].

The effects of surface topography on the behaviour of

a thin-film flow have long been the subject of intensive studies. Numerous research efforts have been devoted to analysing the surface roughness effects in hydrodynamic lubrication by averaging the film thickness or by averaging the flow. The average flow model, developed by Patir and Cheng [6, 7], has the advantage of being easy to apply to the distributed roughness problem. The average flow rate is expressed as the rate of flow passing through the averaged spacing, multiplied by the flow factors. The flow factors are obtained previously by numerical calculation using a small control area exhibiting surface roughness. However, several critical arguments about the physical interpretation and calculation of the flow factors have been discussed intensively. Elrod [8] tried to generalize the flow factors analytically. Hu and Zheng [9] revealed the effects of boundary conditions, grid systems and the reason why the calculated results for the flow factors obtained by different researchers widely differed from each other. The flow factor tensor expression proposed by Tripp [1] was the most complete description of the interaction between the lubricant flow and the roughness. He outlined the mathematical method using the perturbation approach with a Green function technique. The flow factor tensors are symmetrical but not necessarily

diagonal according to the coordinate system. When the principal directions of the surface textures are not parallel to the coordinate axes, the extra flow corresponding to the cross flow factor in one direction might be large enough to affect the flow in another direction. The cross flow factors introduce a channelling or pumping action [10] brought about by either the sliding motion or the pressure gradient on the sides of the bearing. Lo [10] further explored this area and discussed the effects of roughness orientation on flow factors by using a mapping and double scale analysis. His results showed that the maximum pumping action takes place when the asperities are oriented at 45° to the direction of pressure gradient or sliding motion. Those results, though, are limited to incompressible lubricants and surfaces with the same periodic anisotropic roughness.

In recent years, the effects of surface roughness on compressible lubricating films have been studied even more intensively as the flying head spacing in magnetic storage devices becomes smaller [11–18]. Crone *et al* [17] and Makino *et al* [18] independently extended the roughness flow factors for the average Reynolds equation under conditions of rarefied gas lubrication. The effects of molecular mean free path on the roughness-induced flow factors were taken into account by the assumption that the local compressibility is small. This assumption ensured that the fluctuation in the flow factors would be insignificant compared with the fluctuations in the pressure and the film thickness. The derivation of flow factors was carried out by means of the perturbation approach with a Green function technique [1]. The results were presented in terms of average Knudsen number, Peklenik number and the ratio of mean film thickness to standard deviation of composite roughness height. However, the roughness orientations are assumed to be parallel to the sliding direction.

The perturbation technique is adopted in this study, and the flow factors are derived by using the mapping function. The generalized gas lubrication model [5] for advanced sliders is used to correct the rarefaction effects. The flow factors can be applied to rarefied gas lubrication for two surfaces with given roughness characteristics (including standard deviations of roughness heights, roughness orientation angles and Peklenik numbers). Finally, detailed derivations and the static analysis of a magnetic recording head flying above a rotating disc under the ultra-thin-spacing condition are discussed.

2. The derivation

A fluid film between two surfaces, z_1 and z_2 , moving in the x - y plane with arbitrary translational velocities u_1 and u_2 , measured relative to a point in the fluid remote from the film region, is illustrated in figure 1. The random variables, δ_1 and δ_2 , represent a stationary stochastic process distributed with standard deviations σ_1 and σ_2 , respectively, about the zero mean. Under the assumption that the two variables, δ_1 and δ_2 , are uncorrelated, the standard deviation, σ , of the roughness

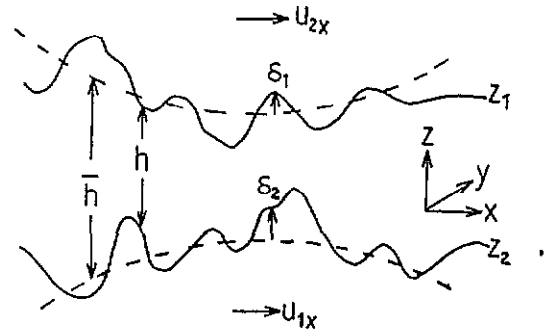


Figure 1. A schematic diagram of the rough air bearing interface.

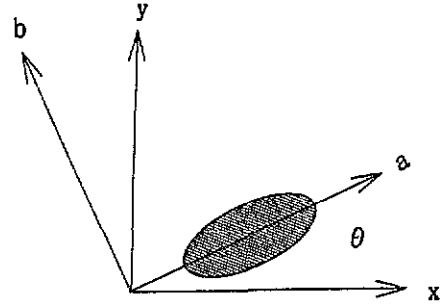


Figure 2. A schematic diagram of a representative asperity with orientation angle θ .

combination $(\delta_2 - \delta_1)$ is given simply by $(\sigma_1^2 + \sigma_2^2)^{1/2}$. In figure 2, the orientation variables, θ_1 and θ_2 , are the angles between principal directions of the representative asperity in each surface and the x axis.

Under the slip flow regime, the modified Reynolds equation can be written as

$$\partial_i \left(\frac{1}{12\mu} \psi_P(p, h) \rho h^3 \partial_i p - U_i \rho h \right) = \partial_t(\rho h) \quad (1)$$

with $\partial_i = \partial/\partial x_i$ and $\partial_t = \partial/\partial t$. Here, $\psi_P(p, h)$ is the modification to the total mass flow rate due to rarefaction effects, μ is the coefficient of viscosity, ρ is the density, p is the pressure and h is the local film thickness of the lubrication film. In the following derivation, the overbars denote the nominal or expected values.

Assuming that the density or molecular mean free path in the control volume (figure 1) is nearly constant, the incompressible Reynolds equation is applicable, using an Euler transformation to interchange the time and space derivatives of h , and is given by

$$\partial_i \left(\frac{1}{12\mu} \psi_P(\bar{p}, h) \rho h^3 \partial_i p \right) = -V_i \partial_i Z \quad (2)$$

with local film thickness

$$h = z_2 - z_1 = \bar{h} + \delta_2 - \delta_1 = \bar{h}(1 + \lambda_-)$$

mean surface height with non-dimensional roughness coordinates $\lambda_{\pm} = (\delta_2 \pm \delta_1)/\bar{h}$ (small parameters)

$$Z = \frac{1}{2}(z_2 + z_1) = \frac{1}{2}(\delta_2 + \delta_1) = \frac{1}{2}\bar{h}\lambda_+$$

slip surface velocity in the i direction

$$V_i = u_{2i} - u_{1i}$$

and mean surface velocity in the i direction

$$U_i = \frac{1}{2}(u_{2i} + u_{1i}).$$

Tripp [1] assumed that the pressure gradient in equation (2) could be replaced by a reduced pressure gradient, and that the reduced pressure could be expressed in terms of a perturbation expansion in a small parameters ($\varepsilon^n q_n$), namely

$$\frac{1}{\mu} \partial_i p = \frac{1}{\mu_0} \partial_i q \quad (3)$$

$$q = \sum_{n=0}^{\infty} \varepsilon^n q_n. \quad (4)$$

On substituting equations (3) and (4) into equation (2), the first two lowest-order equations for q_0 and εq_1 are found to be

$$\partial_i \partial_i q_0 = 0 \quad (5)$$

$$\varepsilon \partial_i \partial_i q_1 = -f(\bar{p}, \bar{h}) \partial_i (\lambda_- \partial_i q_0) - V_i \frac{6\mu_0 \bar{h}}{F(\bar{p}, \bar{h})} \partial_i \lambda_+ \quad (6)$$

where $F(\bar{p}, h) = \psi_P(\bar{p}, h) h^3$ and

$$f(\bar{p}, h) = \frac{\bar{h}}{F(\bar{p}, \bar{h})} \left. \frac{dF(\bar{p}, h)}{dh} \right|_{h=\bar{h}}.$$

Furthermore, the solutions of equations (5) and (6) are

$$\partial_i q_0 = \text{constant} \quad (7)$$

$$\varepsilon q_1 = -f(\bar{p}, \bar{h}) \partial_i q_0 \iint G(x_i; x_i^*) \partial_i^* \lambda_- dx_i^* - \frac{6\mu_0 \bar{h}}{F(\bar{p}, \bar{h})} V_i \iint G(x_i; x_i^*) \partial_i^* \lambda_+ dx_i^* \quad (8)$$

where

$$G(x_i; x_i^*) = \frac{1}{4\pi} \ln[(x_i - x_i^*)(x_i - x_i^*)].$$

For thin film lubrication, the mass flow rate in the i directions is

$$Q_i = -\frac{\bar{p}}{12\mu RT} \psi_P(\bar{p}, h) h^3 \partial_i p + \frac{\bar{p}}{RT} h U_i \quad (9)$$

and the average mass flow rate is expressed as

$$\bar{Q}_i = -\frac{\bar{p}}{12\mu RT} \psi_P(\bar{p}, \bar{h}) \bar{h}^3 \phi_{ij}^p \partial_j \bar{p} + \frac{\bar{p}}{RT} \left(h U_i - \frac{\sigma}{2} \phi_{ij}^s V_j \right) \quad (10)$$

where ϕ_{ij}^p and ϕ_{ij}^s are the pressure and shear flow tensors, and σ is the standard deviation of composite roughness.

On comparing equation (10) with the average value of equation (9), the average pressure flow becomes

$$-\frac{\bar{p}}{12\mu RT} \psi_P(\bar{p}, h) h^3 \partial_i p = -\frac{\bar{p}}{12\mu RT} \psi_P(\bar{p}, \bar{h}) \bar{h}^3 \phi_{ij}^p \partial_j \bar{p} - \frac{\bar{p}}{RT} \frac{\sigma}{2} \phi_{ij}^s V_j. \quad (11)$$

By using equation (4) and retaining the first three lowest-order terms, the average pressure flow rate in the x direction can also be represented as

$$-\frac{\bar{p}}{12\mu_0 RT} F(\bar{p}, \bar{h} + \bar{h} \lambda_-) \partial_i \left(\sum_{n=0}^{\infty} \varepsilon^n q_n \right) \approx -\frac{\bar{p}}{12\mu_0 RT} F(\bar{p}, \bar{h}) \left[\partial_i \bar{q}_0 + \left(\frac{\sigma}{\bar{h}} \right)^2 g(\bar{p}, \bar{h}) \partial_i \bar{q}_0 + f(\bar{p}, \bar{h}) \varepsilon \lambda_- \partial_i q_1 \right] \quad (12)$$

From equations (8) and (12), the expected term $\varepsilon \lambda_- \partial_i q_1$ can be expressed as

$$\varepsilon \lambda_- \partial_i q_1 = -f(p, h) \partial_j q_0 \times \iint \partial_i G(x; x^*) \partial_j^* [\lambda_-(x) \lambda_-(x^*)] dx^* - \frac{6\mu_0 \bar{h}}{F(\bar{h})} V_j \iint \partial_i G(x; x^*) \partial_j^* [\lambda_-(x) \lambda_+(x^*)] dx^*. \quad (13)$$

Likewise, the expected values in the right side of equation (13) can be represented as

$$\overline{\lambda_-(x, y) \lambda_-(x^*, y^*)} = \left(\frac{\sigma}{\bar{h}} \right)^2 \left[\left(\frac{\sigma_2}{\sigma} \right)^2 r_2 + \left(\frac{\sigma_1}{\sigma} \right)^2 r_1 \right] \quad (14)$$

$$\overline{\lambda_-(x, y) \lambda_+(x^*, y^*)} = \left(\frac{\sigma}{\bar{h}} \right)^2 \left[\left(\frac{\sigma_2}{\sigma} \right)^2 r_2 - \left(\frac{\sigma_1}{\sigma} \right)^2 r_1 \right] \quad (15)$$

where r_1 and r_2 are the autocorrelation functions of each surface. Here, the cross correlation function between the two surfaces is assumed to be zero.

Usually, a pressure gradient in the x direction can produce a flow in the y direction under certain conditions of roughness anisotropy. The pressure and shear flow tensors are diagonal only when one of the principal directions of the representative asperity is coincident with the x direction, and the two surfaces are uncorrelated. For two uncorrelated surfaces, any anisotropic surface in the a - b coordinate system can be transformed into the x - y system by the following mapping function:

$$x = a \cos \theta - b \sin \theta \quad (16)$$

$$y = a \sin \theta + b \cos \theta. \quad (17)$$

In this paper, we choose Tripp's elliptic form autocorrelation function at (a^*, b^*) as

$$r = \exp\{-[\alpha(a - a^*)^2 + \beta(b - b^*)^2]\} \quad (18)$$

where $(\beta/\alpha)^{1/2}$ corresponds to the Peklenik parameter γ . This parameter γ is defined as the ratio of the a and b half correlation lengths, where the a and b axes coincide with the principal directions of the representative asperity. Thus the values $\gamma = 0, 1$ and ∞ represent transverse, isotropic and longitudinal roughness, respectively.

Finally, applying the mapping functions, equations (16) and (17), the integration of equation (13) can be calculated in the a - b coordinate system. The flow factors can be obtained from equation (11). Detailed derivations are shown in appendix A.

3. Results

The flow factors are derived functions of the Peklenik numbers, γ_i , the standard deviations, σ_i , and the roughness orientations, θ_i , of each surface, where $i = 1, 2$. They are also functions of film thickness ratio $H_s = \bar{h}/\sigma$, defined as the ratio of average film thickness to the standard deviation of composite roughness, and the inverse Knudsen number, D . In the following list of flow factors, overbars are neglected for convenience of presentation:

$$\phi_{ij}^p = \phi_{ij}^p(\gamma_1, \gamma_2, \theta_1, \theta_2, \sigma_1, \sigma_2, H_s, D) \quad (19)$$

$$\phi_{ij}^s = \phi_{ij}^s(\gamma_1, \gamma_2, \theta_1, \theta_2, \sigma_1, \sigma_2, H_s, D) \quad (20)$$

$$\phi_{xx}^p = 1 + g \left(\frac{\sigma}{\bar{h}} \right)^2 \left\{ 1 - \frac{f^2}{g} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 \Phi_2 + \left(\frac{\sigma_1}{\sigma} \right)^2 \Phi_1 \right] \right\} \quad (21)$$

$$\phi_{xy}^p = \phi_{yx}^p = -f \left(\frac{\sigma}{\bar{h}} \right)^2 \left[\left(\frac{\sigma_2}{\sigma} \right)^2 \Psi_2 + \left(\frac{\sigma_1}{\sigma} \right)^2 \Psi_1 \right] \quad (22)$$

$$\phi_{yy}^p(\theta_1, \theta_2) = \phi_{xx}^p(90^\circ - \theta_1, 90^\circ - \theta_2) \quad (23)$$

$$\phi_{xx}^s = -f \frac{\sigma}{\bar{h}} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 \Phi_2 - \left(\frac{\sigma_1}{\sigma} \right)^2 \Phi_1 \right] \quad (24)$$

$$\phi_{xy}^s = \phi_{yx}^s = -f \frac{\sigma}{\bar{h}} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 \Psi_2 - \left(\frac{\sigma_1}{\sigma} \right)^2 \Psi_1 \right] \quad (25)$$

$$\phi_{yy}^s(\theta_1, \theta_2) = \phi_{xx}^s(90^\circ - \theta_1, 90^\circ - \theta_2) \quad (26)$$

where Φ_i and Ψ_i are the orientation coefficients of the i th surface. The coefficients are expressed as

$$\begin{aligned} \Phi_i &= \cos^2 \theta_i \frac{1}{\gamma_i + 1} + \sin^2 \theta_i \frac{\gamma_i}{\gamma_i + 1} \\ &= \frac{1}{2} + \frac{1}{2} \frac{1 - \gamma_i}{1 + \gamma_i} \cos(2\theta_i) \end{aligned} \quad (27)$$

$$\begin{aligned} \Psi_i &= \sin \theta_i \cos \theta_i \left(\frac{1}{\gamma_i + 1} - \frac{\gamma_i}{\gamma_i + 1} \right) \\ &= \frac{1}{2} \frac{1 - \gamma_i}{1 + \gamma_i} \sin(2\theta_i) \end{aligned} \quad (28)$$

where f and g are shown in appendix B.

To verify the accuracy of the present model, the special cases without roughness orientations, namely $\theta_1 = \theta_2 = 0^\circ$, are considered. In such cases, we obtain $\Phi_i = 1/(\gamma_i + 1)$ and $\Psi_i = 0$, and the flow factors

$$\begin{aligned} \phi_{xx}^p &= 1 + g \left(\frac{\sigma}{\bar{h}} \right)^2 \\ &\times \left\{ 1 - \frac{f^2}{g} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 \frac{1}{\gamma_2 + 1} + \left(\frac{\sigma_1}{\sigma} \right)^2 \frac{1}{\gamma_1 + 1} \right] \right\} \end{aligned} \quad (29)$$

$$\phi_{yy}^p(\gamma_1, \gamma_2) = \phi_{xx}^p \left(\frac{1}{\gamma_1}, \frac{1}{\gamma_2} \right) \quad (30)$$

$$\phi_{xx}^s = -f \frac{\sigma}{\bar{h}} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 \frac{1}{\gamma_2 + 1} - \left(\frac{\sigma_1}{\sigma} \right)^2 \frac{1}{\gamma_1 + 1} \right] \quad (31)$$

$$\phi_{yy}^s(\gamma_1, \gamma_2) = \phi_{xx}^s \left(\frac{1}{\gamma_1}, \frac{1}{\gamma_2} \right) \quad (32)$$

$$\phi_{xy}^p = \phi_{yx}^p = \phi_{xy}^s = \phi_{yx}^s = 0. \quad (33)$$

The above results are identical to the results of Makino *et al* [18].

As the inverse Knudsen number, D , increases, the rarefaction effect decreases, and as D becomes very large, from equations (B1) and (B2), we have $f \rightarrow 3$, and $g \rightarrow 3$. Thus we have the following expression of flow factors for continuous flow:

$$\begin{aligned} \phi_{xx}^p &= 1 + 3 \left(\frac{\sigma}{\bar{h}} \right)^2 \\ &\times \left\{ 1 - 3 \left[\left(\frac{\sigma_2}{\sigma} \right)^2 \frac{1}{\gamma_2 + 1} + \left(\frac{\sigma_1}{\sigma} \right)^2 \frac{1}{\gamma_1 + 1} \right] \right\} \end{aligned} \quad (34)$$

$$\phi_{xx}^s = 3 \frac{\sigma}{\bar{h}} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 \frac{1}{\gamma_2 + 1} - \left(\frac{\sigma_1}{\sigma} \right)^2 \frac{1}{\gamma_1 + 1} \right] \quad (35)$$

which are exactly the same as the results of Tripp [1].

Furthermore, using the flow factors, equations (29)–(33), and the operational conditions in the report by Makino *et al* [18], the numerical calculation of load ratio for the two-sided roughness case is shown in figure 3, which reveals a very good agreement with the results of Makino *et al*.

For the convenience of discussion, the cases of moving roughness ($\sigma_1 \neq 0, \sigma_2 = 0$) and stationary roughness ($\sigma_1 = 0, \sigma_2 \neq 0$) are considered. The oriented asperities enhance the flow in the x direction due to there being a negative pressure gradient in that direction for the case $\phi_{xx}^p > 1$, and restrict the flow for the case $\phi_{xx}^p < 1$. The cross pressure flow factor exists for anisotropic asperities with their principal directions not parallel to or normal to the sliding directions. On the other hand, for an isotropic surface or anisotropic asperities whose principal directions coincide with the x - y coordinates, the cross pressure flow factors vanish. For a positive orientation angle, a negative pressure gradient in the x direction will channel a flow in the y direction. Likewise, for a negative orientation angle, the results are reversed. Because of the dead volume

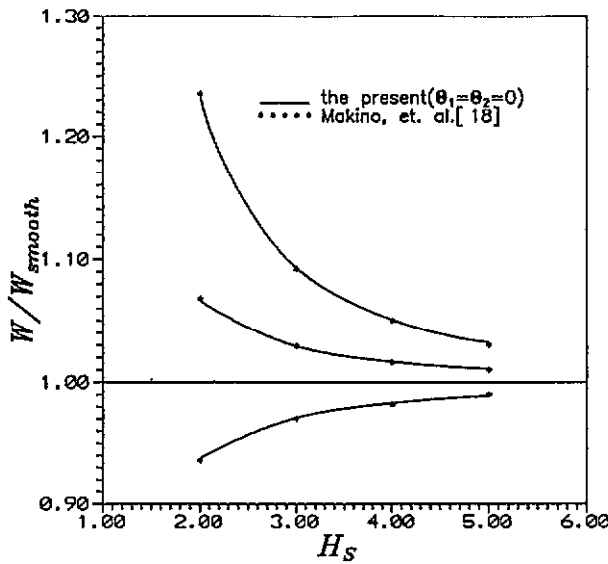


Figure 3. A comparison with the results of Makino *et al* [18].

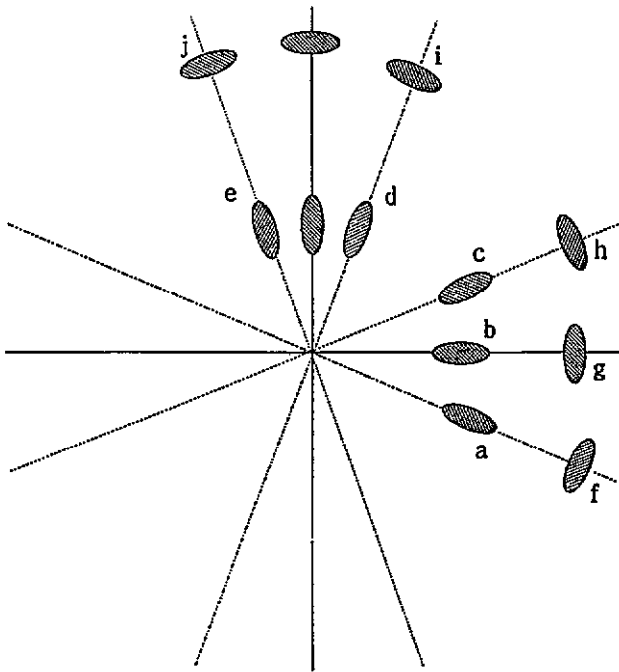


Figure 4. The identity of asperities.

and orientation of asperities in the lubricating film, the restriction and channelling of the pressure flow for the moving roughness case are the same as that of the stationary roughness case. For the two-sided roughness case, the pressure flow factors ϕ_{ij}^p may be viewed as the combined effect of the orientation coefficient multiplied by the weightings $(\sigma_i/\sigma)^2$ of the two surfaces.

The shear flow factor ϕ_{xx}^s is negative for the stationary roughness case and positive for the moving roughness case. Because the moving asperities trap the fluid, the flow is enhanced if the rough surface is moving, and is restricted if the rough surface is stationary. For moving roughness, a scraping in the x direction induces a pushing flow in the y direction for negative orientation angles, while for stationary roughness, the same orientation angle will result in channelling flow

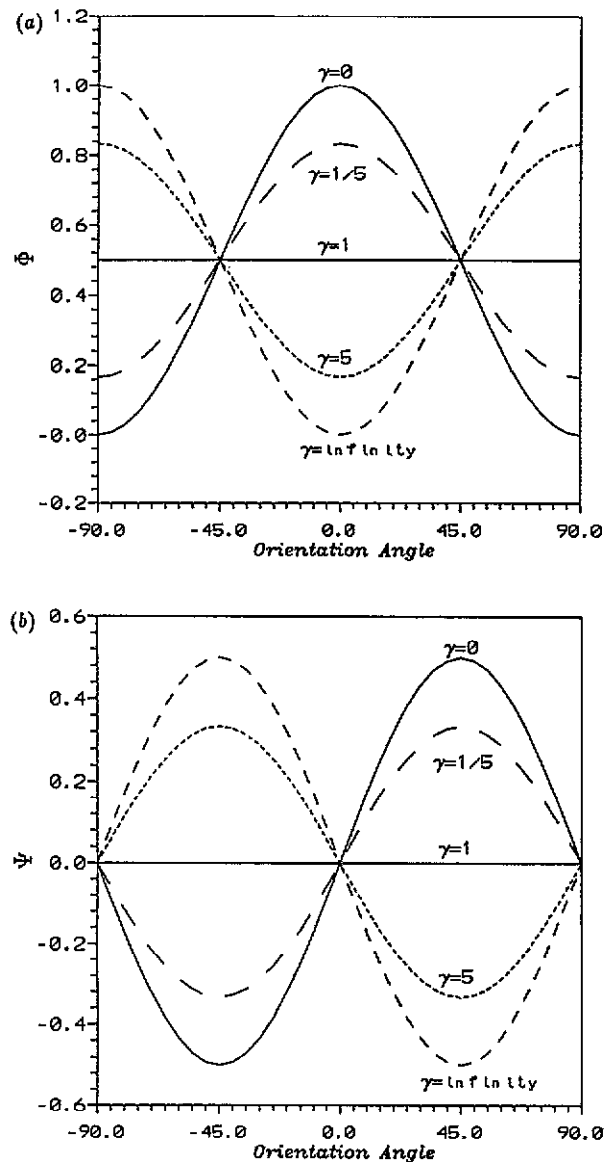


Figure 5. (a) The relationship between orientation coefficient, Φ , and orientation angle, θ , for various Peklenik numbers. (b) The relationship between orientation coefficient, Ψ , and orientation angle, θ , for various Peklenik numbers.

in the negative y direction. Owing to the opposite effects of moving roughness and stationary roughness cases, the shear flow factors ϕ_{ij}^s for two-sided roughness can be viewed as the difference between the orientation coefficients multiplied by the weighting $(\sigma_i/\sigma)^2$ of the two surfaces. If two surfaces have the same orientation coefficients ($\gamma_1 = \gamma_2, \theta_1 = \theta_2$) and weightings ($\sigma_1 = \sigma_2$), or have isotropic roughness, then the shear flow factors ϕ_{ij}^s are all zero.

Since the flow factors represent the combined effects of the orientation coefficients of each surfaces with weighting $(\sigma_i/\sigma)^2$, the properties of the orientation coefficients are discussed as follows.

(i) Identity properties

$$\Phi(\gamma, \theta) = \Phi(1/\gamma, 90^\circ + \theta) + \Phi(\gamma, 180^\circ + \theta)$$

$$\Psi(\gamma, \theta) = \Psi(1/\gamma, 90^\circ + \theta) + \Psi(\gamma, 180^\circ + \theta). \quad (36)$$

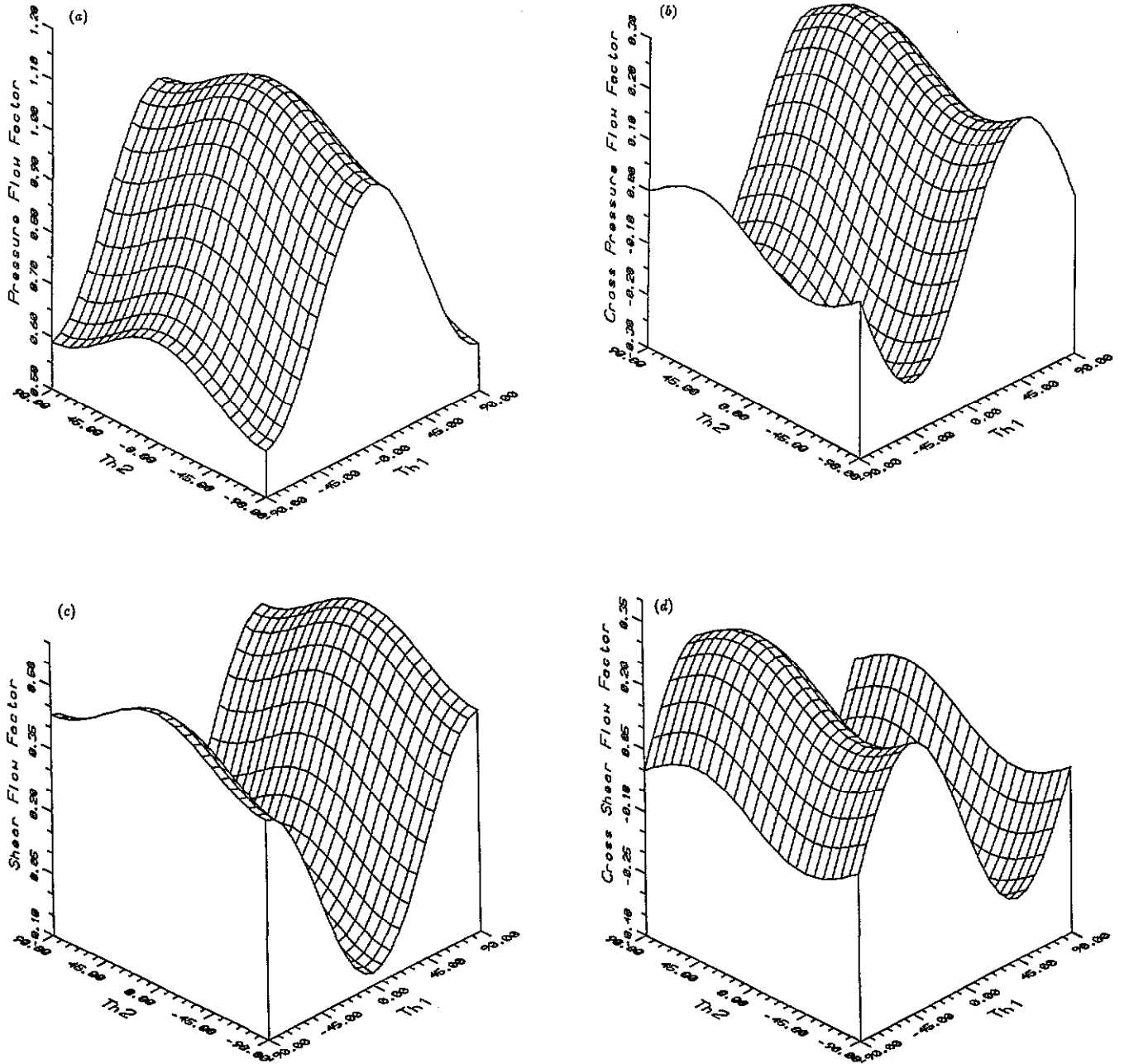


Figure 6. (a) The relationship between pressure flow factor, ϕ_{xx}^p , and orientation angles, θ_1 and θ_2 , of bearing surfaces. (b) The relationship between cross pressure flow factor, ϕ_{xy}^p , and orientation angles, θ_1 and θ_2 , of bearing surfaces. (c) The relationship between shear flow factor, ϕ_{xx}^s , and orientation angles, θ_1 and θ_2 , of bearing surfaces. (d) The relationship between cross shear flow factor, ϕ_{xy}^s , and orientation angles, θ_1 and θ_2 , of bearing surfaces.

A representative asperity is plotted in figure 4, in which $a = i$, $c = j$ and so forth.

(ii) Isotropic roughness ($\gamma = 1$)

$$\Phi(1, \theta) = \frac{1}{2} \quad \Psi(1, \theta) = 0. \quad (37)$$

Because the isotropic properties are independent of the orientation angle, the orientation coefficients are constant for any orientation angle.

(iii) Symmetric properties

$$\Phi(\gamma, \theta) = \Phi(\gamma - \theta)$$

$$\Psi(\gamma, \theta) = \Psi(\gamma, 90^\circ - \theta). \quad (38)$$

(iv) Skew symmetric properties

$$\Phi(\gamma, \theta) - \frac{1}{2} = -[\Phi(\gamma, -\theta) - \frac{1}{2}]$$

$$\Psi(\gamma, \theta) = -\Psi(\gamma, -\theta) = -\Psi(1/\gamma, \theta) \quad (39)$$

or

$$\Phi(\gamma, \theta) - \Phi(1, \theta) = -[\Phi(\gamma, -\theta) - \Phi(1, \theta)]$$

$$\begin{aligned} \Psi(\gamma, \theta) - \Psi(1, \theta) &= -[\Psi(\gamma, -\theta) - \Psi(1, \theta)] \\ &= -\Psi(1/\gamma, \theta). \end{aligned} \quad (39')$$

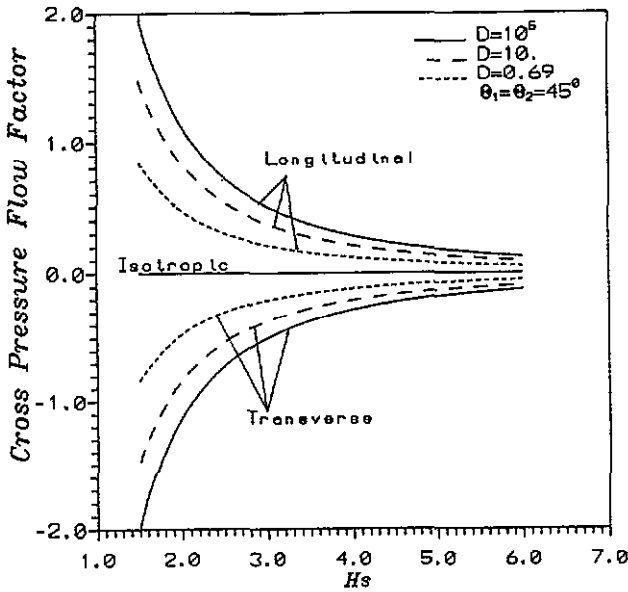


Figure 7. The relationship between cross pressure flow factors, ϕ_{xy}^p , and film ratio, H_s , for various inverse Knudsen numbers.

Additionally, the orientation coefficients, Φ and Ψ , are plotted as functions of the orientation angle, θ , for various Peklenik numbers (see figures 5(a) and (b)). The coefficient Φ is symmetric about $\theta = 0^\circ, \pm 90^\circ$ and skew symmetric about $\theta = \pm 45^\circ$. Conversely, the coefficient Ψ is symmetric about $\theta = \pm 45^\circ$ and skew symmetric about $\theta = 0^\circ, \pm 90^\circ$.

If two surfaces have the same Peklenik number but different weighting, then the flow factors vary as the orientation angles of the two surfaces do. From figure 6(a)–(d), the flow factors are expressed as functions of the orientation angles, θ_1 and θ_2 , for the conditions that $D = 0.69$, $H_s = 2.4$, $\gamma_1 = \gamma_2 = 5$ and $\sigma_1 = 2\sigma_2$. The flow factors vary sinusoidally with the orientation angles and have properties similar to those of Φ and Ψ . The variation in the θ_1 direction is larger than that in the θ_2 direction due to the large weighting, $(\sigma_1/\sigma_2)^2$, of surface 1.

For the case in which $\theta_1 = \theta_2 = 45^\circ$, $\Phi_1 = \Phi_2 = \frac{1}{2}$ and

$$\Psi_1 = \Psi_2 = \frac{1}{2} \frac{1 - \gamma}{1 + \gamma}$$

ϕ_{xx}^p and ϕ_{yy}^p are independent of the Peklenik number γ , while ϕ_{xy}^p and ϕ_{yx}^p are dependent on it. Though the diagonal flow factors are the same, the off-diagonal flow factors are not the same. In figure 7, the cross pressure flow factors are plotted as functions of H_s for various inverse Knudsen numbers ($D = 10^5, 10$ and 0.69) and various Peklenik numbers (longitudinal, isotropic and transverse roughness). Because the rarefaction effect reduces the roughness effect, the values of flow factors are closer to those of the smooth cases for lower D values. As can also be seen in figure 7, the longitudinal roughness and transverse roughness cases have the opposite effect on cross pressure flow factors, that is $\phi_{xy}^p(\gamma, \theta) = -\phi_{yx}^p(1/\gamma, \theta)$. For an isotropic surface, the cross pressure flow factors and the cross shear flow factors are all zero.

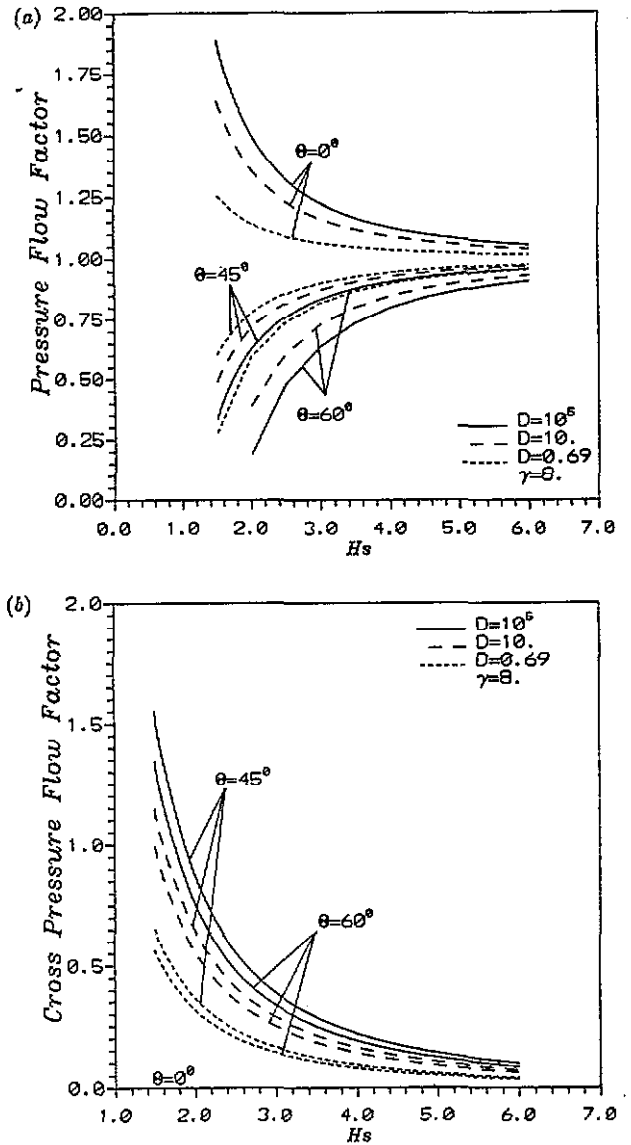


Figure 8. (a) The relationship between pressure flow factor, ϕ_{xx}^p , and film ratio, H_s , for two-sided roughness with $\sigma_1 = \sigma_2$ and $\gamma_1 = \gamma_2 = \gamma$. (b) The relationship between cross pressure flow factor, ϕ_{xy}^p , and film ratio, H_s , for two-sided roughness with $\sigma_1 = \sigma_2$ and $\gamma_1 = \gamma_2 = \gamma$.

In figure 8(a) and (b), the pressure flow factors for two-sided roughness with $\sigma_1 = \sigma_2$ and $\gamma_1 = \gamma_2 = \gamma$ are plotted as functions of H_s for various orientation angles ($\theta_1 = \theta_2 = \theta = 0^\circ, 45^\circ$ and 60°) and various inverse Knudsen numbers. The roughness enhances the flow ($\phi_{xx}^p > 1$) for $\theta = 0^\circ$, while it restricts the flow ($\phi_{xx}^p < 1$) for $\theta \geq 45^\circ$. The channelling flow due to pressure gradient in the perpendicular direction is maximum at $\theta = 45^\circ$ and is minimum at $\theta = 0^\circ$ and 90° .

Under the slip flow regime, the generalized average Reynolds equation for the finite slider (taper-flat slider) is written as

$$\partial_i \left(\bar{\phi}_{ij}^p \psi_p \frac{\bar{h}^3}{12\mu} \partial_j \bar{p} \right) = -V_j \partial_i \left(Z \delta_{ij} + \frac{1}{2} \sigma \bar{\phi}_{ij}^s \right). \quad (40)$$

The finite-difference scheme, with the Gauss-Seidel over-relaxation method, is applied to solve equation (40) numerically. The accuracy of results is checked by

doubling the mesh density, and the error is less than 1%. Convergence is achieved until $|(G_{i,j}^{n+1} - G_{i,j}^n)/G_{i,j}^n| < 10^{-5}$ at all nodes. Typical operation conditions based on fixed configuration are as follows:

- (i) slider length $l = 0.5$ mm,
- (ii) slider width $b = 0.0515$ mm,
- (iii) taper length $x_t = 0.05$ mm,
- (iv) taper height $h_t = 0.4356$ μm ,
- (v) minimum film thickness ratio $h_0 = 0.05$ μm ,
- (vi) pitch angle $\theta_0 = 0.000202$,
- (vii) modified bearing number $\Lambda_b = 12$,
- (viii) the inverse Knudsen number $D_0 = 0.69$,
- (ix) the standard deviation of composite roughness $\sigma = 0.025$ μm ,
- (x) the lubricant (air under standard atmospheric conditions) and
- (xi) accommodation coefficients of the boundary surface ($\alpha = 1$, assuming diffuse reflection).

For the slender sliders in this analysis, the side flow is dominant. For the case $\theta_1 = \theta_2 = 0^\circ$, the roughness of longitudinal type ($\gamma > 1$) restricts the side flow, the roughness of transverse type ($\gamma < 1$) enhances the side flow and the isotropic roughness ($\gamma = 1$) has little influence on the side flow. Recall that a restriction of flow in one direction will result in an enhancement of flow in the perpendicular direction, since the restriction of flow in one direction encourages pressure generation but the enhancement of flow reduces it. As the orientation angles of the two surfaces change from 0° , the cross flow factors (ϕ_{xy}^p and ϕ_{xy}^s) appear. As shown in figure 9 ($\sigma_1 = \sigma_2$, $\gamma_1 = \gamma_2 = 9$ and $H_s = 2.0$), the variation in load capacity in the θ_1 direction is more significant than that in the θ_2 direction. This is because surface 1 is the moving surface, and the fluid contained between asperities of the moving surface will be pushed and channelled. Furthermore, the roughness of the stationary surface (surface 2) will restrict and channel the flow. With regard to the orientation effects on load capacity, the contribution of the moving surface is larger than that of the stationary surface. The three-dimensional plot shows that the load capacity is largest when $\theta_1 = \theta_2 = 0^\circ$ and is lowest when $\theta_1 = \pm 90^\circ$ and $\theta_2 = \pm 90^\circ$.

In figure 10(a), the load capacities for two-sided roughness ($\sigma_1 = \sigma_2$, $\theta_1 = \theta_2$) are plotted as functions of H_s for various orientations and various inverse Knudsen numbers. The load capacities for moving roughness and stationary roughness plotted as functions of H_s are shown in figure 10(b) and (c). Also, the load capacities for two-sided roughness with $\sigma_1 = 2\sigma_2$ are shown in figure 10(d). As the H_s value increases, the roughness effect decreases and the load capacities approach the smooth case value. The load capacity for two-sided roughness can be viewed as the combination of the load capacities for moving roughness and stationary roughness, multiplied by the weighting $(\sigma_i/\sigma)^2$. As the orientation angles increase simultaneously, the representative asperities change from roughness of longitudinal type to roughness of transverse

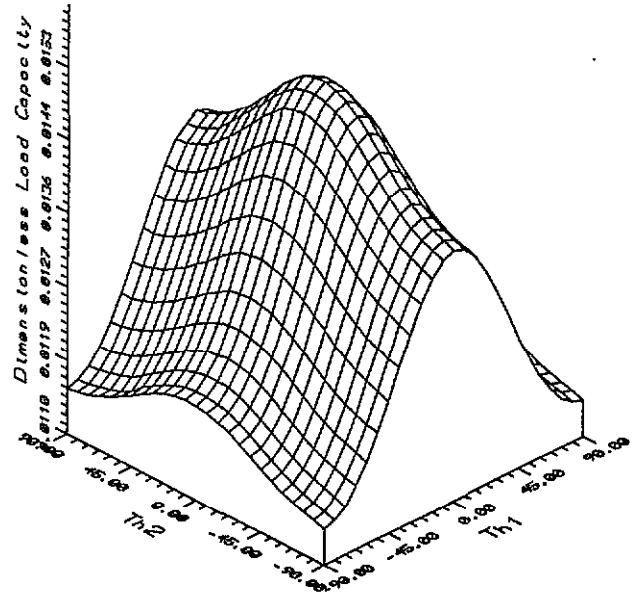


Figure 9. The relationship between non-dimensional load capacity, W , and orientation angles, θ_1 and θ_2 , of bearing surfaces.

type, and thus the load capacities decrease. The increase in inverse Knudsen number D decreases the rarefaction effects and increases the load capacity. These results have already been discussed in several papers [2, 3, 5], so we will not discuss them in this paper.

4. Conclusion

This study has presented a comprehensive derivation for modifying flow factors, taking into consideration the combined effects of individual surface characteristics (orientation angles, Peklenik numbers and derivations of roughness heights). The effects of rarefaction on roughness flow factors were considered under the assumption of small local compressibility. Two orientation coefficients were presented in order to reveal the combined effects of the Peklenik number and orientation angle. The flow factors presented in this paper are applicable to two-dimensional random moving roughness with given orientation angles and Peklenik numbers for the two boundaries in thin film gas lubrication. Some of the results obtained were the following.

(i) The explicit forms of flow factors were derived and expressed as functions of the roughness orientations, the Peklenik numbers, the standard deviations of roughness heights of each surface, the inverse Knudsen number and the film thickness ratio of the lubrication film.

(ii) Two orientation coefficients were proposed in order to reveal the combined effects of Peklenik number and orientation angle, and the properties of these orientation coefficients are discussed. Additional efforts, though, are needed to defined proper parameters for surface characteristics.

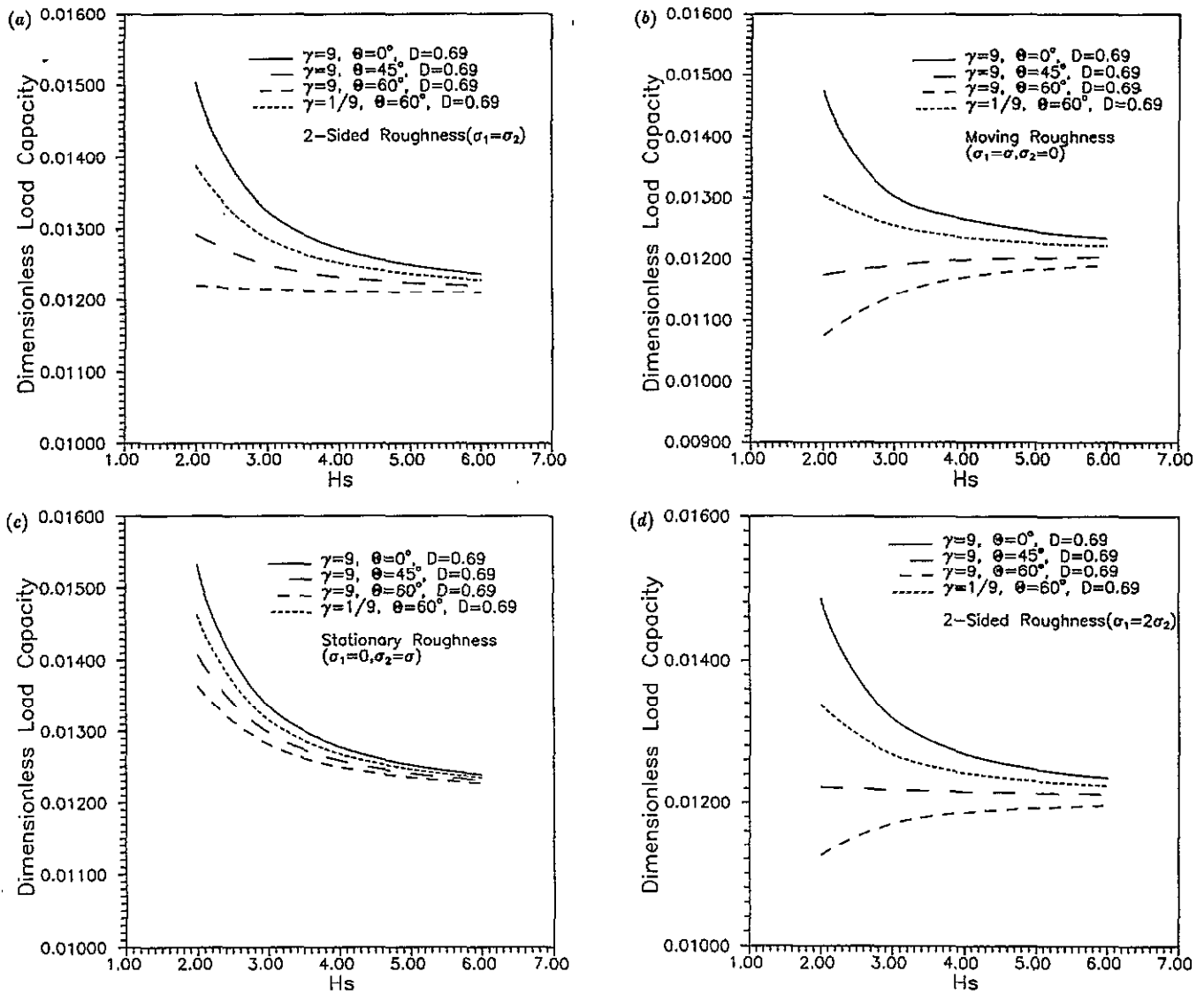


Figure 10. (a) The relationship between non-dimensional load capacity, W , and film ratio, H_s , for the two-sided roughness case with $\sigma_1 = \sigma_2$ and $\theta_1 = \theta_2$. (b) The relationship between non-dimensional load capacity, W , and film ratio, H_s , for the moving roughness case. (c) The relationship between non-dimensional load capacity, W , and film ratio, H_s , for the stationary roughness case. (d) The relationship between non-dimensional load capacity, W , and film ratio, H_s , for the two-sided roughness case with $\sigma_1 = 2\sigma_2$ and $\theta_1 = \theta_2$.

(iii) When the asperities are oriented at 45° to the sliding direction, the pressure flow factors (ϕ_{xx}^p) are the same but the cross pressure flow factors (ϕ_{xy}^p) are different.

(iv) The change in flow factors is significant as the orientation coefficients vary (namely, there are changes in orientation angle and Peklenik number).

(v) The effect of surface characteristics of the moving surface on load capacity is more significant than that of the stationary surface.

In recent developments, rotary actuators have been used to reduce the size of the disc drive, and the area of the slider is reduced to decrease the flying height. Under such conditions, the yawing effect becomes important. Thus the consideration of effects of roughness orientation on the static performance is necessary when the roughness characteristics of sliders

and discs are known beforehand.

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Appendix. A

The integrations in equation (13) are listed in the following. By the use of coordinate transformation, the transformations about surface 1 and surface 2 are

$$\begin{aligned}\frac{\partial}{\partial x} &= \cos \theta_1 \frac{\partial}{\partial a_1} - \sin \theta_1 \frac{\partial}{\partial b_1} = \cos \theta_2 \frac{\partial}{\partial a_2} - \sin \theta_2 \frac{\partial}{\partial b_2} \\ \frac{\partial}{\partial y} &= \sin \theta_1 \frac{\partial}{\partial a_1} + \cos \theta_1 \frac{\partial}{\partial b_1} = \sin \theta_2 \frac{\partial}{\partial a_2} + \cos \theta_2 \frac{\partial}{\partial b_2}\end{aligned}\quad (A1)$$

with autocorrelation functions

$$\begin{aligned} r_1 &= \exp\{-[\alpha_1(a_1 - a_1^*)^2 + \beta_1(b_1 - b_1^*)^2]\} \\ r_2 &= \exp\{-[\alpha_2(a_2 - a_2^*)^2 + \beta_2(b_2 - b_2^*)^2]\} \end{aligned} \quad (A2)$$

and Peklenik numbers

$$\gamma_1 = (\beta_1/\alpha_1)^{1/2} \quad \gamma_2 = (\beta_2/\alpha_2)^{1/2}. \quad (A3)$$

The integration terms on the right-hand side of equation (13) are

$$\begin{aligned} & \iint \partial_i G(x; x^*) \partial_j^* [\lambda_-(x) \lambda_+(x^*)] dx^* \\ &= \iint \partial_i G(x; x^*) \partial_j^* \left[\left(\frac{\sigma_2}{\sigma} \right)^2 r_2 \pm \left(\frac{\sigma_1}{\sigma} \right)^2 r_1 \right] dx^* \\ &= \iint \left(\frac{\partial G}{\partial x} \frac{\partial}{\partial x^*} + \frac{\partial G}{\partial x} \frac{\partial}{\partial y^*} + \frac{\partial G}{\partial y} \frac{\partial}{\partial x^*} + \frac{\partial G}{\partial y} \frac{\partial}{\partial y^*} \right) \\ & \quad \times \left[\left(\frac{\sigma_2}{\sigma} \right)^2 r_2 \pm \left(\frac{\sigma_1}{\sigma} \right)^2 r_1 \right] dx^* dy^*. \end{aligned} \quad (A4)$$

Furthermore, the first integration term on the right-hand side of the above equation is

$$\begin{aligned} & \iint \frac{\partial G}{\partial x} \frac{\partial}{\partial x^*} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 r_2 \pm \left(\frac{\sigma_1}{\sigma} \right)^2 r_1 \right] dx^* dy^* \\ &= \left(\frac{\sigma_2}{\sigma} \right)^2 \iint \left(\cos \theta_2 \frac{\partial}{\partial a_2} - \sin \theta_2 \frac{\partial}{\partial b_2} \right) \frac{1}{4\pi} \\ & \quad \times \ln[(a_2 - a_2^*)^2 + (b_2 - b_2^*)^2] \left(\cos \theta_2 \frac{\partial}{\partial a_2^*} - \sin \theta_2 \frac{\partial}{\partial b_2^*} \right) \\ & \quad \times \exp\{-[\alpha_2(a_2 - a_2^*)^2 + \beta_2(b_2 - b_2^*)^2]\} da_2^* db_2^* \\ & \quad \pm \left(\frac{\sigma_1}{\sigma} \right)^2 \iint \left(\cos \theta_1 \frac{\partial}{\partial a_1} - \sin \theta_1 \frac{\partial}{\partial b_1} \right) \frac{1}{4\pi} \\ & \quad \times \ln[(a_1 - a_1^*)^2 + (b_1 - b_1^*)^2] \left(\cos \theta_1 \frac{\partial}{\partial a_1^*} - \sin \theta_1 \frac{\partial}{\partial b_1^*} \right) \\ & \quad \times \exp\{-[\alpha_1(a_1 - a_1^*)^2 + \beta_1(b_1 - b_1^*)^2]\} da_1^* db_1^* \\ &= \left(\frac{\sigma_2}{\sigma} \right)^2 \left(\cos^2 \theta_2 \frac{1}{\gamma_2 + 1} + \sin^2 \theta_2 \frac{\gamma_2}{\gamma_2 + 1} \right) \\ & \quad \pm \left(\frac{\sigma_1}{\sigma} \right)^2 \left(\cos^2 \theta_1 \frac{1}{\gamma_1 + 1} + \sin^2 \theta_1 \frac{\gamma_1}{\gamma_1 + 1} \right). \end{aligned} \quad (A5)$$

Similarly,

$$\begin{aligned} & \iint \frac{\partial G}{\partial x} \frac{\partial}{\partial y^*} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 r_2 \pm \left(\frac{\sigma_1}{\sigma} \right)^2 r_1 \right] dx^* dy^* \\ &= \iint \frac{\partial G}{\partial y} \frac{\partial}{\partial x^*} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 r_2 \pm \left(\frac{\sigma_1}{\sigma} \right)^2 r_1 \right] dx^* dy^* \\ &= \left(\frac{\sigma_2}{\sigma} \right)^2 \sin \theta_2 \cos \theta_2 \left(\frac{1}{\gamma_2 + 1} - \frac{\gamma_2}{\gamma_2 + 1} \right) \\ & \quad \pm \left(\frac{\sigma_1}{\sigma} \right)^2 \sin \theta_1 \cos \theta_1 \left(\frac{1}{\gamma_1 + 1} - \frac{\gamma_1}{\gamma_1 + 1} \right) \end{aligned} \quad (A6)$$

$$\begin{aligned} & \iint \frac{\partial G}{\partial y} \frac{\partial}{\partial y^*} \left[\left(\frac{\sigma_2}{\sigma} \right)^2 r_2 \pm \left(\frac{\sigma_1}{\sigma} \right)^2 r_1 \right] dx^* dy^* \\ &= \left(\frac{\sigma_2}{\sigma} \right)^2 \left(\cos^2 \theta_2 \frac{\gamma_2}{\gamma_2 + 1} + \sin^2 \theta_2 \frac{1}{\gamma_2 + 1} \right) \\ & \quad \pm \left(\frac{\sigma_1}{\sigma} \right)^2 \left(\cos^2 \theta_1 \frac{\gamma_1}{\gamma_1 + 1} + \sin^2 \theta_1 \frac{1}{\gamma_1 + 1} \right). \end{aligned} \quad (A7)$$

The integration limits are from minus infinity to plus infinity.

Table 1. The rarefaction coefficients.

	C_0	C_1	C_2	C_3
$5 \leq D$	1.0	6.0972	6.3918	-12.8124
$0.15 \leq D \leq 5$	0.8311	7.5052	0.9391	-0.05814
$0.01 \leq D \leq 0.15$	-13.3751	12.6403	0.0991	-0.000416

Appendix. B

The rarefaction coefficients, $\Psi_p(D) = \sum_{k=0}^3 C_k D^{-k}$, were derived by Fukui and Kaneko [5] with C_k coefficients proposed to fit the non-dimensional flow based on the linearized Boltzmann equation. The coefficients are given in table 1.

The coefficient functions of rarefaction f and g [18] are

$$f(D) = \sum_{k=0}^2 \frac{(3-k)C_k D^{-k}}{\Psi_p(D)} \quad (B1)$$

$$g(D) = (3C_0 + C_1 D^{-1})/\Psi_p(D) \quad (B2)$$

with $D = \sqrt{\pi}/(2K_n)$ the inverse Knudsen number and $K_n = \lambda/h$ the Knudsen number.

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