

Lubrication of Long Porous Slider Bearings*

(Use of the Brinkman-Extended Darcy Model)

Chi-Chuan HWANG**, Jaw-Ren LIN***
and Rong-Fuh YANG****

We apply the Brinkman-extended Darcy model to analyze the hydrodynamic lubrication of long porous slider bearings. The solutions are obtained and compared with those based on the Darcy model. The results show that the effects of viscous shear given by the Brinkman-extended Darcy model lead to a higher load capacity and a lower friction parameter. It is found that a thicker porous bearing gives a greater effect of viscous shear on load capacity and friction parameter.

Key Words: Hydrodynamic Lubrication, Porous Slider Bearings

1. Introduction

Due to the low cost and no need for any exterior oil supply once the bearing with porous material impregnated with oil are installed, the self-lubricated porous slider bearings have been extensively used in industrial applications for a long time. Traditionally, the analyses of the hydrodynamic lubrication of porous slider bearings have been based on Darcy's law where shearing stresses in the oil film are neglected, such as in works by Murti⁽¹⁾, Wang⁽²⁾, Kumar⁽³⁾, Sanni and Ayomidele⁽⁴⁾. However, in a theoretical analysis for axi symmetric flow through a porous wall into a narrow, uniform gap, Ishizawa and Hori⁽⁵⁾ indicated that the flow generally has a tangential velocity component in the porous wall substance, and they verified

theoretically the existence of a slip of fluid on the porous wall surface. Furthermore, studies by Beavers and Joseph⁽⁶⁾ and Beavers et al.⁽⁷⁾ showed experimental results in which the effects of viscous shear on the fluid layer penetrating into the permeable matrix are observed. Beavers and Joseph⁽⁶⁾ retained the Darcy model and proposed a slip velocity boundary condition at the interface to account for the viscous effect, and proposed the dimensionless slip coefficient, α , which depends on the variation of the porosity. From their results, the value of $\alpha=0.1$ is used for aloxite porous specimens, and the value of $\alpha=0.78\sim 4.01$ is used for foam metal porous specimens. Therefore, Murti⁽⁸⁾ applied the slip velocity boundary condition to narrow porous journal bearings at the film-bearing interface. Bhatt and Sacheti⁽⁹⁾ used the slip velocity boundary condition to comment on the work by Wang⁽²⁾. Kaneko⁽¹⁰⁾ examined the static and dynamic characteristics of short porous journal bearings with anisotropic permeability using the slip velocity condition. Kaneko and Doi⁽¹¹⁾ also applied the slip velocity condition to analyze the static and dynamic characteristics of short porous journal bearings with nonuniform permeability. Recently, other works by Neale

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** Department of Mechanical Engineering, Chung Yuan University, Chung Li, Taiwan 32023, R. O. C.

*** Department of Mechanical Engineering, Nanya Junior College, Chung Li, Taiwan 32024, R. O. C.

**** Graduate Student, Department of Mechanical Engineering, Chung Yuan University, Chung Li, Taiwan 32023, R. O. C.

With the following assumptions,

- (a) the film is thin;
 - (b) there is no variation of pressure across the fluid film;
 - (c) there are no external forces acting on the film;
 - (d) the fluid inertia is small compared to the viscous shear;
 - (e) the fluid is an incompressible Newtonian lubricant having constant properties;
 - (f) the flow is laminar,
- the reduced Navier-Stokes equations for the oil film are

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

$$\frac{\partial p}{\partial y} = 0 \quad (3)$$

$$\frac{\partial p}{\partial z} = \mu \frac{\partial^2 w}{\partial y^2} \quad (4)$$

The continuity equation is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (5)$$

The corresponding boundary conditions are

$$u(x, h, z) = U \quad (6)$$

$$v(x, h, z) = V \quad (7)$$

$$w(x, h, z) = 0, \quad (8)$$

where U and V are respectively the surface velocity and the normal velocity of the slider.

Moreover, with the following assumptions,

- (a) the porous bearing material is homogeneous and isotropic;
- (b) the flow of oil in the bearing is laminar;
- (c) the variation of pressure across the porous medium is also negligible, the Brinkman-extended Darcy equations in the porous medium are given by

$$\frac{\partial \bar{p}}{\partial x} = -\frac{\mu}{K} \bar{u} + \tilde{\mu} \frac{\partial^2 \bar{u}}{\partial y^2} \quad (9)$$

$$\frac{\partial \bar{p}}{\partial y} = 0 \quad (10)$$

$$\frac{\partial \bar{p}}{\partial z} = -\frac{\mu}{K} \bar{w} + \tilde{\mu} \frac{\partial^2 \bar{w}}{\partial y^2}, \quad (11)$$

The continuity equation in the porous medium is

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0, \quad (12)$$

where $\bar{u}$, $\bar{v}$ and $\bar{w}$ are the Darcian velocity components.

The boundary conditions for the oil in the porous medium are

$$\bar{u}(x, -H, z) = 0 \quad (13)$$

$$\bar{v}(x, -H, z) = 0 \quad (14)$$

$$\bar{w}(x, -H, z) = 0. \quad (15)$$

The pressure is continuous at the interface, therefore

$$p(x, 0, z) = \bar{p}(x, 0, z). \quad (16)$$

Since velocities and viscous shear stresses are continuous at the interface, then

$$u(x, 0, z) = \bar{u}(x, 0, z) \quad (17)$$

$$w(x, 0, z) = \bar{w}(x, 0, z) \quad (18)$$

$$v(x, 0, z) = \bar{v}(x, 0, z) \quad (19)$$

$$\mu \frac{\partial u}{\partial y} \Big|_{y=0} = \tilde{\mu} \frac{\partial \bar{u}}{\partial y} \Big|_{y=0} \quad (20)$$

$$\mu \frac{\partial w}{\partial y} \Big|_{y=0} = \tilde{\mu} \frac{\partial \bar{w}}{\partial y} \Big|_{y=0}. \quad (21)$$

Integrating Eqs. (2) and (9) with respect to y , and using conditions (3), (6), (10), (13), (16), (17), and (20), we obtain the velocity components u and $\bar{u}$ as

$$u = g_1(y)U + \frac{1}{2\mu} \frac{\partial p}{\partial x} \{y^2 - h^2 g_1(y) + Kg_2(y)\} \quad (23)$$

and

$$\bar{u} = g_3(y)U + \frac{1}{2\mu} \frac{\partial p}{\partial x} \{-h^2 g_3(y) + \alpha h K^{1/2} g_4(y) + Kg_5(y)\}, \quad (24)$$

where $g_1(y) \sim g_5(y)$ are expressed in Appendix A. Furthermore, integrating equations (4) and (11) with respect to y , and using conditions (3), (8), (10), (15), (16), (18), and (21), we obtain the velocity components w and $\bar{w}$ as

$$w = \frac{1}{2\mu} \frac{\partial p}{\partial z} \{y^2 - h^2 g_1(y) + Kg_2(y)\} \quad (25)$$

and

$$\bar{w} = \frac{1}{2\mu} \frac{\partial p}{\partial z} \{-h^2 g_3(y) + \alpha h K^{1/2} g_4(y) + Kg_5(y)\}. \quad (26)$$

The modified Reynolds equation can be obtained by integrating the two continuity equations. First we make use of continuity equation (5). By replacing u and w with their respective expressions, and integrating $\partial v/\partial y$ with respect to y with the conditions of $v = v(x, 0, z)$ at $y=0$ and condition (7), we obtain one Reynolds equation for the oil film. By the same procedure for continuity equation (12), and applying the conditions of $\bar{v} = \bar{v}(x, 0, z)$ at $y=0$ and condition (14), we also obtain another Reynolds equation for the porous medium. Now, utilizing velocity continuity condition (19), we can combine the above two Reynolds equations into the final modified Reynolds equation,

$$\frac{\partial}{\partial x} \left(h_a^3 A^* \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left(h_a^3 A^* \frac{\partial p}{\partial z} \right) = 6\mu U(1+B^*) \frac{\partial h}{\partial x}, \quad (27)$$

where A^* and B^* are shown in Appendix B.

2.2 Long bearing approximation

Introducing the dimensionless variables and parameters

$$x^* = \frac{x}{a},$$

$$z^* = \frac{z}{b},$$

$$p^* = p \frac{h_a^2}{\mu a U},$$

$$\lambda = \frac{b}{a},$$

$$\Phi = \frac{KH}{h_a^3},$$

into modified Reynolds equation (27) yields

$$\frac{\partial}{\partial x^*} \left(A^* \frac{\partial p^*}{\partial x^*} \right) + \frac{1}{\lambda^2} \frac{\partial}{\partial z^*} \left(A^* \frac{\partial p^*}{\partial z^*} \right) = 6(1 + B^*) \frac{\partial h^*}{\partial x^*}. \quad (28)$$

For a long bearing, $\lambda \gg 1$, and the term is reduced

$\frac{1}{\lambda^2} \frac{\partial}{\partial z^*} \left(A^* \frac{\partial p^*}{\partial z^*} \right)$ can be neglected. Since $h^* = h^*(x^*)$, Eq. (28) reduces to

$$\frac{d}{dx^*} \left(A^* \frac{dp^*}{dx^*} \right) = 6(1 + B^*) \frac{dh^*}{dx^*} \quad (29)$$

The boundary conditions for the lubricant in the film region are

$$p^* = 0 \text{ at } x^* = 0 \quad (30)$$

$$p^* = 0 \text{ at } x^* = 1. \quad (31)$$

2.3 Dimensionless film pressure

Integrating Eq. (29) twice with respect to θ , we get

$$p^* = \int \frac{\int 6(1 + B^*) dh^*}{A^*} dx^* + c_1 \int \frac{1}{A^*} dx^* + c_2, \quad (32)$$

where c_1 and c_2 are integration constants. Applying the boundary conditions given above, we can obtain numerically the dimensionless pressure distributions for the oil film. Thus, all the bearing characteristics can now be determined.

2.4 Bearing characteristics

2.4.1 Load capacity The load capacity is obtained by integrating the film pressure $x^* = 0$ to $x^* = 1$. The load capacity per unit length W is given by

$$W = a \int_{x^*=0}^{x^*=1} p dx^*. \quad (33)$$

In terms of dimensionless quantities, the load capacity yields

$$W^* = \frac{W}{\mu U} \frac{h_a^2}{a^2} = \int_0^1 p^* dx^*. \quad (34)$$

2.4.2 Coefficient of friction The shear stress τ_s acting on the slider is

$$\tau_s = \mu \left. \frac{\partial u}{\partial y} \right|_{y=h}. \quad (35)$$

The friction force acting on the slider per unit length is

$$F = \int_0^1 \tau_s a dx^* = a \int_0^1 \mu \left. \frac{\partial u}{\partial y} \right|_{y=h} dx^*. \quad (36)$$

The coefficient of friction $f = F/W$ is obtained from

$$f \left(\frac{a}{h_a} \right) = - \frac{h_a}{W^* (h_0 - h_a)} \ln \frac{\alpha \sigma_0 h_a + h_a}{\alpha \sigma_0 h_0 + h_a} + \frac{1}{W^*} \int_0^1 \frac{\alpha \sigma_0^2 h^{*2} + 2 \sigma_0 h^* + 2 \alpha \left(\frac{dp^*}{dx^*} \right)}{2 \sigma_0 (\alpha \sigma_0 h^* + 1)} dx^*. \quad (37)$$

Table 1 Dimensionless film pressure
 $h_a/a = 0.0025$, $h_0/a = 0.005$, $H/a = 0.1$, $\Phi = 0.001$

	$\alpha = 0.78$	$\alpha = 1.00$	$\alpha = 1.45$	$\alpha = 4.01$
X^*	P^*	P^*	P^*	P^*
0.00	.0000	.0000	.0000	.0000
0.10	.0362	.0368	.0374	.0385
0.20	.0702	.0713	.0726	.0747
0.30	.1008	.1204	.1043	.1073
0.40	.1263	.1283	.1307	.1345
0.50	.1447	.1470	.1498	.1542
0.60	.1531	.1555	.1585	.1631
0.70	.1479	.1503	.1531	.1576
0.80	.1247	.1266	.1290	.1328
0.90	.0776	.0788	.0803	.0826
1.00	.0000	.0000	.0000	.0000

Table 2 Dimensionless film load capacity
 $h_a/a = 0.0025$, $h_0/a = 0.005$, $H/a = 0.1$

	$\alpha = 0.78$	$\alpha = 1.00$	$\alpha = 1.45$	$\alpha = 4.01$
Φ	W^*	W^*	W^*	W^*
0.0001	.1581	.1583	.1584	.1587
0.0002	.1578	.1580	.1582	.1586
0.0003	.1574	.1577	.1580	.1584
0.0006	.1568	.1572	.1576	.1581
0.0010	.1560	.1564	.1569	.1576
0.0018	.1547	.1553	.1559	.1569
0.0032	.1528	.1535	.1544	.1556
0.0056	.1498	.1508	.1519	.1535
0.0100	.1454	.1466	.1479	.1499
0.0178	.1387	.1402	.1418	.1442
0.0316	.1291	.1307	.1326	.1354
0.0562	.1159	.1176	.1196	.1226
0.1000	.0992	.1008	.1027	.1057
0.1778	.0801	.0814	.0830	.0856
0.3162	.0606	.0615	.0627	.0647
0.5623	.0429	.0434	.0441	.0455
1.0000	.0286	.0287	.0290	.0298

3. Results and Discussion

3.1 Effect of α

According to Neale and Nader⁽¹²⁾, the Brinkman-extended Darcy model with constant porosity is identical to the slip flow model if the slip coefficient is identified as α , the square root of $(\tilde{\mu}/\mu)$. The effect of the value of α on the film pressure distributions and the bearing characteristics is presented in Tables 1 and 2 and Fig. 2. Table 1 presents the film pressure under different values of α . Tables 2 shows the effect of the variations of α on the load capacity. They show that the higher value of α causes a higher value of the film pressure and load capacity. However, the effect of the variations of α on P^* and W^* is small and negligible. Figure 2 shows the coefficient of friction under different values of α . It is found that higher values of α yield higher friction force. From the above,

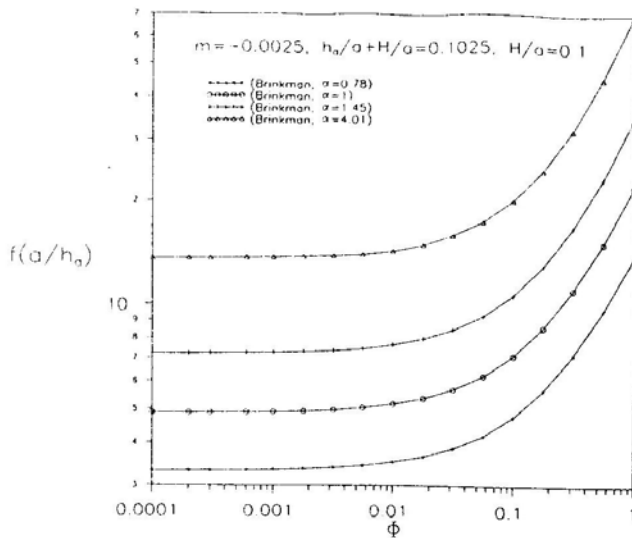


Fig. 2 Friction parameter at different values of α

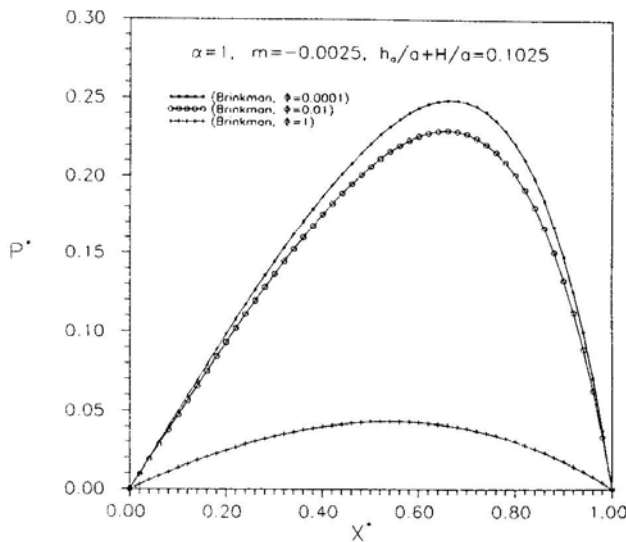


Fig. 3 Dimensionless film pressure distributions at different values of permeability parameter

the effect of the variations of α on P^* and W^* for different porous media is found to be small and can be neglected, but not on $f(a/h_a)$. Without losing generality, we adopt the value of $\alpha=1$, which is suggested by Neale and Nader⁽¹²⁾, for the following evaluation.

3.2 Film pressure distribution

Dimensionless film pressure distributions along x^* at different values of the permeability parameter Φ are shown in Fig. 3. It is found that when the value of Φ increases, the pressure decreases. This is because of larger Φ for lower viscous shear effect in the porous matrix.

Figure 4 shows the dimensionless film pressure distributions at different values of Φ resulting from the Brinkman-extended Darcy model and the Darcy model. The results based on the Darcy model were obtained by Murti⁽¹¹⁾. We can see that the Brinkman-extended Darcy model results in a higher film pressure

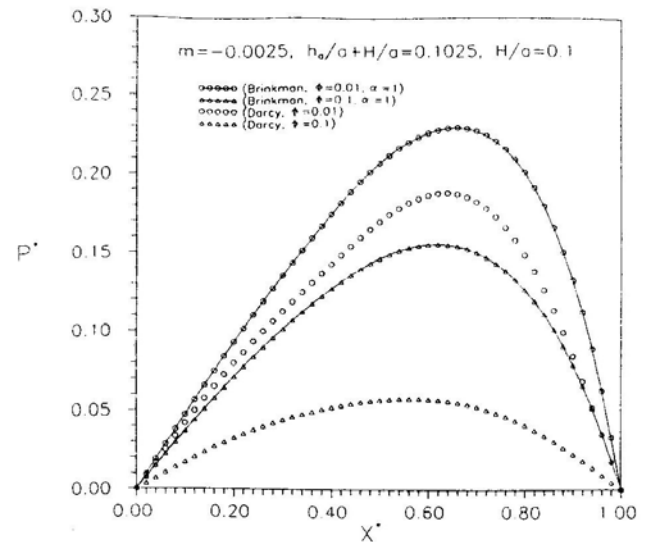


Fig. 4 Comparison of dimensionless film pressure distributions at different values of Φ , obtained from the Brinkman-extended Darcy model and the Darcy model

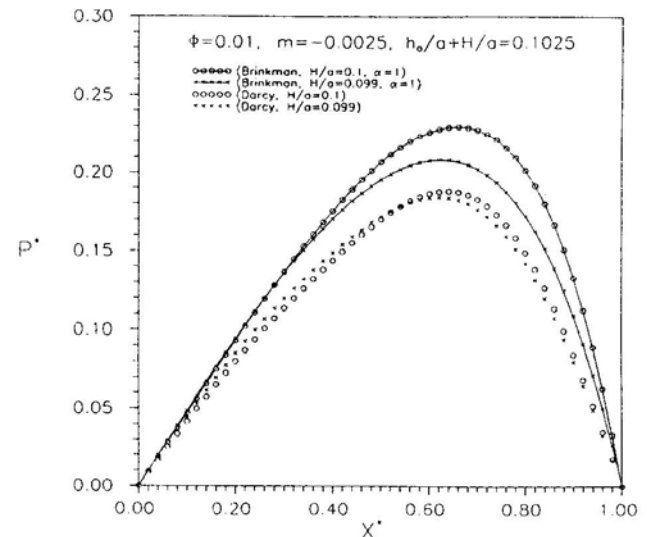


Fig. 5 Comparison of dimensionless film pressure distributions at different values of H/a , obtained from the Brinkman-extended Darcy model and the Darcy model

at $\Phi=0.01$. This can be realized because Brinkman's equations include viscous shear stresses, which are neglected in Darcy's equations. This contributes to the higher film pressure. It can be seen that the viscous shear effects increase when the value of Φ increases.

Figure 5 shows the comparison of dimensionless film pressure distributions at different values of porous wall thickness ratio (H/a), obtained from the Brinkman-extended Darcy model and the Darcy model. Due to the viscous shear effects, the Brinkman-extended Darcy model gives a higher film pressure at $H/a=0.099$. As the porous wall thickness increases, $H/a=0.1$, the viscous shear effects increase.

Figure 6 shows the comparison of dimensionless film pressure distributions at different inclinations (m) of the slider, obtained from the Brinkman-extended Darcy model and the Darcy model. The Brinkman-extended Darcy model results in a higher film pressure at $m = -0.0025$ because of the viscous shear effects. It can be seen that the film pressure increases, i.e., the viscous shear effects increase, when there is greater inclination ($m = -0.0035$) or a higher value of h_0 , the entrance height of oil.

3.3 Load capacity

Figure 7 shows the comparison of the dimensionless load capacities at different values of H/a , which

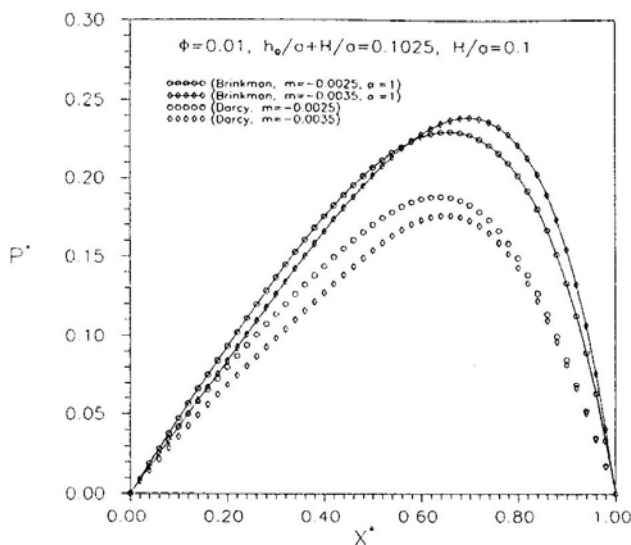


Fig. 6 Comparison of dimensionless film pressure distributions at different inclinations of slider, obtained from the Brinkman-extended Darcy model and the Darcy model

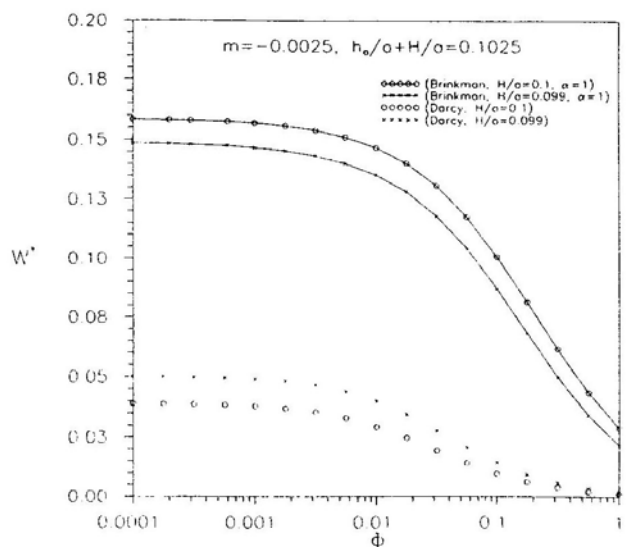


Fig. 7 Comparison of the dimensionless load capacities at different values of H/a , obtained from the Brinkman-extended Darcy model and the Darcy model

were calculated based on the Brinkman-extended Darcy model and the Darcy model. We can see that the Brinkman-extended Darcy model results in a higher load capacity at $H/a = 0.099$. Moreover, we can see that Brinkman's equations include viscous shear stresses, which are neglected in Darcy's equations. The viscous shear effects increase the film pressure, and then increase the load capacity. It is shown that the load capacity increases, i.e., the viscous shear effects increase, when the value of the porous wall thickness ratio increases to $H/a = 0.1$.

Figure 8 shows the comparison of the load capacities at different inclinations (m) of the slider, which were calculated using the Brinkman-extended Darcy model and the Darcy model. It is found that when the inclination of the slider increases, the viscous shear effects on the load capacity increase.

3.4 Friction parameter

Figure 9 shows the comparison of the friction parameters at different values of H/a , which were calculated using the Brinkman-extended Darcy model and the Darcy model. This figure tells us that the Brinkman-extended Darcy model yields a lower friction parameter at $H/a = 0.099$. This is realized because the viscous shear effects increase the load capacity, which appears in the denominator on the right side of Eq. (37). Therefore, the value of the friction parameter is less than that based on the Darcy model. Also, it is found that the viscous shear effects on the friction parameter increase when the porous wall thickness increases.

Figure 10 shows the comparison of the friction parameter at different inclinations (m) of the slider, which was calculated using the Brinkman-extended

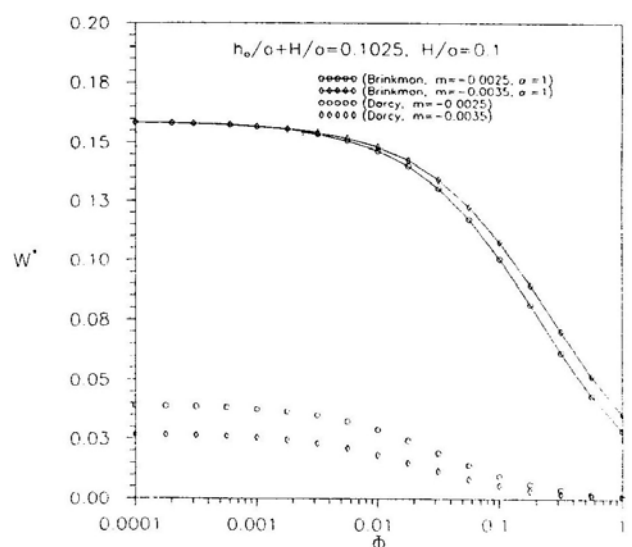


Fig. 8 Comparison of the dimensionless load capacities at different inclinations of slider, obtained from the Brinkman-extended Darcy model and the Darcy model

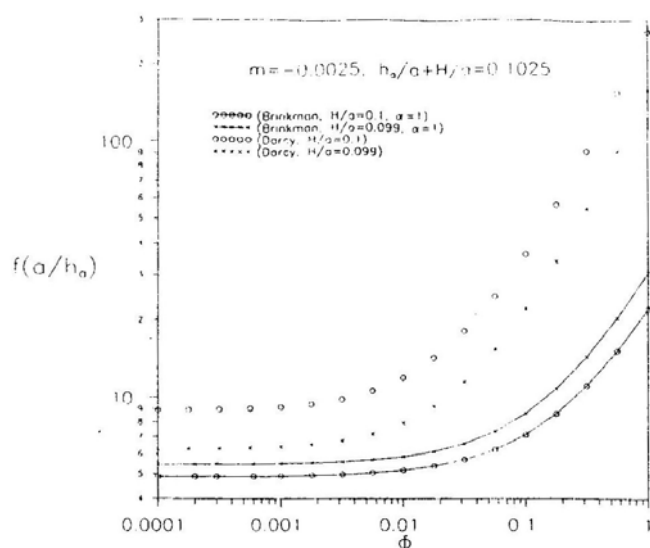


Fig. 9 Comparison of the friction parameters at different values of H/a , obtained from the Brinkman-extended Darcy model and the Darcy model

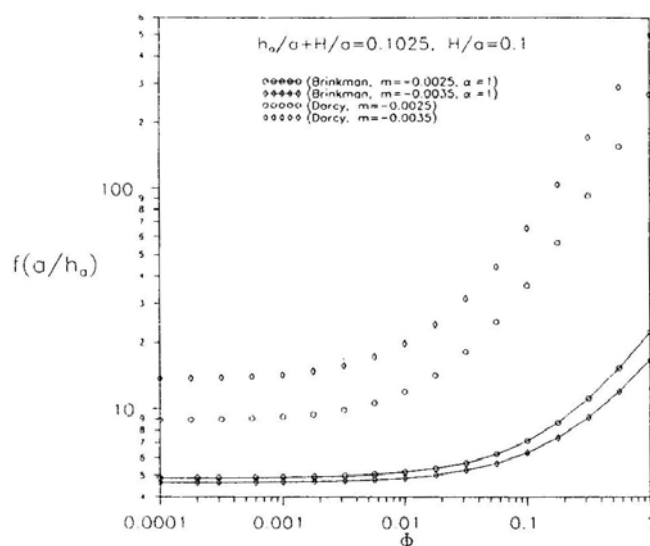


Fig. 10 Comparison of the friction parameters at different inclinations of slider, obtained from the Brinkman-extended Darcy model and the Darcy model

Darcy model and the Darcy model. We can see that when the inclination of the slider increases, the viscous shear effects on the friction parameter increase.

4. Conclusions

Based on the Brinkman-extended Darcy model, we investigate the viscous shear effects on long porous slider bearings. Brinkman's equations, which include viscous shear terms, are applied to obtain a modified Reynolds equation for long porous slider bearings. The solution is numerically obtained for the pressure in the film. Other bearing characteristics such as load capacity and coefficient of friction are also evaluated. The conclusions based on the Brinkman-extended

Darcy model are as follows

(1) The parameter α accounts for the effect of viscous shear in the bearing matrix. For different porous media, the effect of variations of α on the film pressure and the load capacity is small and can be neglected.

(2) Compared with that using the Darcy model, where viscous shear stresses are neglected, the viscous shear effects increase the load capacity, but decrease the friction parameter.

(3) As the value of the porous wall thickness ratio increases, the viscous shear effects increase the load capacity and decrease the friction parameter. Thus, the viscous shear effects on the bearing characteristics progressively increase when we have a thicker porous bearing.

(4) When the inclination of the slider increases, the viscous shear effects increase the load capacity and decrease the friction parameter. Therefore, greater inclination of the slider gives a greater effect of viscous shear on the bearing characteristics.

Other analyses of finite porous slider bearings and dynamic characteristics will be presented in the future.

Appendix A

The expressions of $g_1(y) \sim g_5(y)$

$$g_1(y) = \frac{\left[1 + \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} y + \left[1 - \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} h}{\left[1 + \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} h + \left[1 - \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} y} \quad (\text{A.1})$$

$$g_2 = \frac{\left[1 - \exp\left(-\frac{H}{\alpha K^{1/2}}\right)\right]^2 \frac{2\alpha}{K^{1/2}} (y-h)}{\left[1 + \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} h + \left[1 - \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} y} \quad (\text{A.2})$$

$$g_3(y) = \frac{\exp\left(\frac{y}{\alpha K^{1/2}}\right) - \exp\left(-\frac{y+2H}{\alpha K^{1/2}}\right)}{\left[1 + \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} h + \left[1 - \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} y} \quad (\text{A.3})$$

$$g_4(y) = \frac{2 \left[\exp\left(\frac{y-H}{\alpha K^{1/2}}\right) - \exp\left(-\frac{2H}{\alpha K^{1/2}}\right) + \exp\left(-\frac{y+H}{\alpha K^{1/2}}\right) - 1 \right]}{\left[1 + \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} h + \left[1 - \exp\left(-\frac{2H}{\alpha K^{1/2}}\right)\right] \frac{\alpha}{K^{1/2}} y} \quad (\text{A.4})$$

$$g_5(y) = 2g_3(y)$$

$$\frac{2 \left[\exp\left(\frac{y-H}{\alpha K^{1/2}}\right) - \exp\left(-\frac{2H}{\alpha K^{1/2}}\right) - \exp\left(-\frac{y+H}{\alpha K^{1/2}}\right) + 1 \right]}{\left[1 + \exp\left(-\frac{2H}{\alpha K^{1/2}}\right) \right] \frac{\alpha}{K^{1/2}} h + \left[1 - \exp\left(-\frac{2H}{\alpha K^{1/2}}\right) \right]} \quad (\text{A.5})$$

Appendix B

The expressions of A^* and B^*

$$A^* = h^{*3} + A_1^* h^{*2} + A_2^* h^* + A_3^* - \frac{A_6^*}{A_4^* h^* + A_5^*} \quad (\text{B.1})$$

$$A_1^* = \frac{1 - \exp\left(-\frac{2\sigma}{\alpha}\right)}{1 + \exp\left(-\frac{2\sigma}{\alpha}\right)} \frac{3}{\alpha \sigma_0} \quad (\text{B.2})$$

$$A_2^* = \left\{ 12\alpha^2 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right] \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right) \right]^2 - 3 \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^2 / 1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right\} \times \frac{1}{\alpha^2 \sigma_0^2} \quad (\text{B.3})$$

$$A_3^* = \left\{ 3 \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^3 - 12\alpha^2 \left[1 - \exp\left(-\frac{4\sigma}{\alpha}\right) \right] \times \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right) \right]^2 / \alpha^3 \sigma_0^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^3 \right\} + \left\{ 12\sigma\alpha^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^3 - 12\alpha^4 \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right) \right] \times \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^2 / \alpha^3 \sigma_0^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^3 \right\} \quad (\text{B.4})$$

$$A_4^* = \alpha^3 \sigma_0^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^3 \alpha \sigma_0 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right] \quad (\text{B.5})$$

$$A_5^* = \alpha^3 \sigma_0^3 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^3 \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right) \right] \quad (\text{B.6})$$

$$A_6^* = 24\alpha^4 \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right) \right]^2 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^3 - 12\alpha^4 \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^2 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^2 - 12\alpha^2 \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right) \right]^2 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right] \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^2 + \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^4 \quad (\text{B.7})$$

$$B^* = \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^2 - 2\alpha^2 \left[1 - \exp\left(-\frac{\sigma}{\alpha}\right) \right]^2 \times \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right] / \{ \alpha \sigma_0 \left[1 + \exp\left(-\frac{2\sigma}{\alpha}\right) \right] h^* - \left[1 - \exp\left(-\frac{2\sigma}{\alpha}\right) \right]^2 \} \quad (\text{B.8})$$

$$\sigma = \frac{H}{K^{1/2}} \quad (\text{B.9})$$

$$\sigma_0 = \frac{h_a}{K^{1/2}} \quad (\text{B.10})$$

$$h^* = \frac{h}{h_a} \quad (\text{B.11})$$

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