

A New Modified Reynolds Equation for Ultrathin Film Gas Lubrication

Chi-Chuan Hwang, Rong-Fong Fung, Rong-Fu Yang, Cheng-I Weng, and Wang-Long Li

Abstract—A new modified Reynolds equation for solving the ultrathin film gas lubrication is proposed to overcome the complicated and time-consuming difficulties of solving the linearized Boltzmann equation. The model equation is based on modified high-order slip-flow velocity distribution with three adjustable coefficients, which are corrected according to the Boltzmann model. The results are compared to those obtained using other kinds of currently employed modified Reynolds equations. It shows that the present model produces a closer approximation to that of the exact Boltzmann model than do other models, in a wider range of inverse Knudsen number. In addition, the newly derived equation is widely applicable to practical use.

INTRODUCTION

THE good operation of a computer flying head for magnetic storage strongly depends on the success of gas film lubrication. To increase the recording density of magnetic disk drives, the reduction of spacing between the magnetic head and recording medium is of great importance. Under such reduced spacing (less than $0.1 \mu\text{m}$), rarefied gas effect must be considered. The importance of the rarefaction effect is represented by the Knudsen number, K_n , defined as the ratio of the molecular mean free path λ to spacing h . In this paper, we use the inverse Knudsen number $D (= \sqrt{\pi}h/2\lambda = \sqrt{\pi}/2K_n)$ to represent the importance of the rarefaction effect. Burgdorfer [1] introduced a first-order correction to the continuum non-slip boundary condition used in the Reynolds equation. Hsia and Domoto [2] considered a second-order slip-flow boundary condition for the velocity, and derived a modified Reynolds equation. Furthermore, Mitsuya [3] introduced 1.5-order slip-flow boundary conditions. The above three slip-flow models are accurate only for large inverse Knudsen number. Gans [4], who first derived a lubrication equation using a linearized Boltzmann equation based on the BGK model [5], derived a lubrication equation which is similar to Burgdorfer's first-order modified Reynolds equation; he claimed that this equation should be valid for all inverse Knudsen numbers. Furkui and Kaneko [6] started from a linearized Boltzmann equation

similar to Gans's with different solution method. Consequently, they derived the generalized Reynolds equation including thermal creep flow. Their results showed that the Burgdorfer's first-order slip-flow condition overestimates the load-carrying capacities, while the second-order slip-flow condition underestimates it. To overcome the time-consuming work on the integro-differential equations for Poiseuille flow velocity profiles, Furkui and Kaneko [7] proposed a polynomial fitting procedure to explicitly express Poiseuille flow rate as a function of inverse Knudsen number using cubic polynomials. But the cubic polynomials were not flexible enough to accurately fit the full range of their database. They were, therefore, forced to use three separated cubic polynomials to fit their database. Furthermore, Crone *et al.* [8] present a continued fraction model which approximates the entire database [7] using three parameters. But it expressed well only in the region of $8.86 \times 10^{-3} \leq D \leq 8.86 \times 10^{-1}$.

In this paper, a new simple model for ultrathin film gas lubrication is proposed to overcome the complicated and time-consuming difficulties of solving the linearized Boltzmann equation. To increase the accuracy of results in a wider range of inverse Knudsen number, we model a flow rate based on modified high-order slip-flow velocity distribution with three adjustable coefficients. The adjustable coefficients are corrected according to the linearized Boltzmann model. Based on the corrected velocity, a new modified Reynolds equation is derived. The final corrected flow rate and the numerical results of static load are compared to those of other models for a larger range of inverse Knudsen number.

THEORETIC ANALYSIS

The current modified Reynolds equation was deduced by analogy to the high-order slip-flow model [3]. To increase the accuracy of the present model, an additional correction term with three adjustable coefficients is used in the velocity boundary conditions for the lubrication film, as illustrated in Fig. 1. The velocity boundary conditions and resulting velocity distribution u are modeled and solved as follows:

$$u_{z=0} = U + a \cdot \lambda \left. \frac{\partial u}{\partial z} \right|_{z=0} - \frac{1}{2} \cdot b \cdot D^c \cdot h^2 \cdot \left. \frac{\partial^2 u}{\partial z^2} \right|_{z=0} \quad (1)$$

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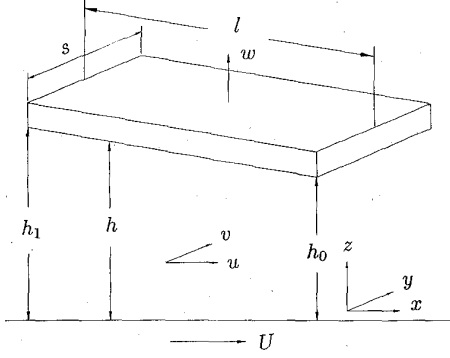


Fig. 1. Coordinate systems and dimensions of lubricating film employed.

$$u_{z=h} = -a \cdot \lambda \left. \frac{\partial u}{\partial z} \right|_{z=h} - \frac{1}{2} \cdot b \cdot D^c \cdot h^2 \cdot \left. \frac{\partial^2 u}{\partial z^2} \right|_{z=h} \quad (2)$$

$$u = U \cdot \left(1 - \frac{z + a \cdot \lambda}{z + 2a \cdot \lambda} \right) - \frac{1}{2\mu} \frac{\partial p}{\partial x} \cdot \{ b \cdot D^c \cdot h^2 + a \cdot h \cdot \lambda + h \cdot z - z^2 \} \quad (3)$$

where μ is the viscosity of the lubrication film, p is the film pressure, h is the local film thickness, λ is the molecular mean free path of air, U is the surface velocity of disk, and the three coefficients a , b , and c are adjustable.

With the velocity distribution for the pressure flow given by the second term in (3), the Poiseuille flow rate between parallel path with the height h is obtained as

$$q_p = \frac{1}{2\mu} \frac{\partial p}{\partial x} h^3 \left\{ b \cdot D^c + \frac{a\sqrt{\pi}}{2D} + \frac{1}{6} \right\}. \quad (4)$$

The present normalized Poiseuille flow rate, Q_P , can be written as

$$Q_P = \frac{q_p}{-h^3/(2\mu D) \cdot (\partial p/\partial x)} = b \cdot D^{1+c} + \frac{a\sqrt{\pi}}{2} + \frac{D}{6}. \quad (5)$$

The three adjustable coefficients a , b , and c given in (5) will be determined by correcting the value of Q_P to match that predicted from the linearized Boltzmann model [6]. The corrected equations are setted, after trial and error, at $D = 10^{-3}$, $D = 10^{-2}$, and $D = 1$; one can gain the optimal three adjustable coefficients. They are

$$\begin{aligned} a &= 0.01807 \\ b &= 1.35355 \\ c &= -1.17468. \end{aligned} \quad (6)$$

In Fig. 2, the effect of inverse Knudsen number on flow rate for various models is compared. The results from the first-order [1], second-order [2], 1.5-order [3] slip-flow models, database [7], continued fraction [8], present

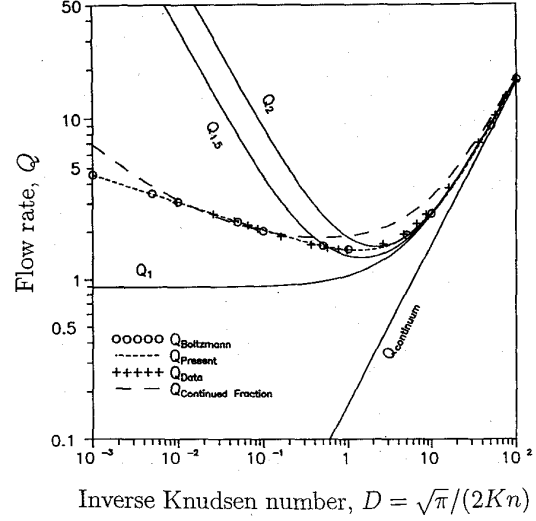


Fig. 2. Comparison of nondimensional flow rate versus inverse Knudsen number resulting from various models.

model, and the Boltzmann model [6] are designated by subscripts 1, 2, 1.5, *Data*, *Continued Fraction*, *Present*, and *Boltzmann*, respectively. It is found that Q_{Present} , Q_{Data} , and $Q_{\text{Continued Fraction}}$ produce a fairly good approximation to $Q_{\text{Boltzmann}}$. While Q_1 underestimates $Q_{\text{Boltzmann}}$, Q_2 overestimates it, and $Q_{1.5}$ approximates $Q_{\text{Boltzmann}}$ closer than Q_2 but still overestimates it. Q_1 , Q_2 , and $Q_{1.5}$ can approximate $Q_{\text{Boltzmann}}$ only for a small region of larger value of inverse Knudsen number D . The database [7] is effective only for $D \geq 0.01$, and three different cubic polynomials for different range of D are needed. None of the three cubic polynomials is flexible enough to fit the exact one in the entire range of D . Although $Q_{\text{Continued Fraction}}$ can fit $Q_{\text{Boltzmann}}$ for a wider range of D , it expresses well only in a small range of D ($8.86 \times 10^{-3} \leq D \leq 8.86 \times 10^{-1}$). These comparisons confirm that the present model can accurately approximate the exact Boltzmann model in the entire range ($10^{-3} \leq D \leq 10^2$).

Insertion of (3) into the continuity equation, and then integration of the equation, results in a modified Reynolds equation in nondimensional form as

$$\frac{\partial}{\partial X} \left[\bar{Q}_p(D) P H^3 \frac{\partial P}{\partial X} \right] + \frac{\partial}{\partial Y} \left[\bar{Q}_p(D) P H^3 \frac{\partial P}{\partial Y} \right] = \Lambda \frac{\partial (P H)}{\partial X} \quad (7)$$

where the ratio $\bar{Q}_p$ can be written by

$$\bar{Q}_p = Q_P / Q_{\text{continuum}} = 1 + 3\sqrt{\pi} \cdot a \cdot D^{-1} + 6b \cdot D^c \quad (8)$$

and $Q_{\text{continuum}} (=D/6)$ is the normalized pressure flow rate with nonslip boundary conditions.

The variables given in (7) are: $P = p/p_a$, the nondimensional pressure (p_a is ambient film pressure); $H = h/h_0$, the nondimensional spacing (h_0 is the minimum flying height); $\Lambda = 6\mu U l / (p_a^2 h_0^2)$, the bearing number (l is

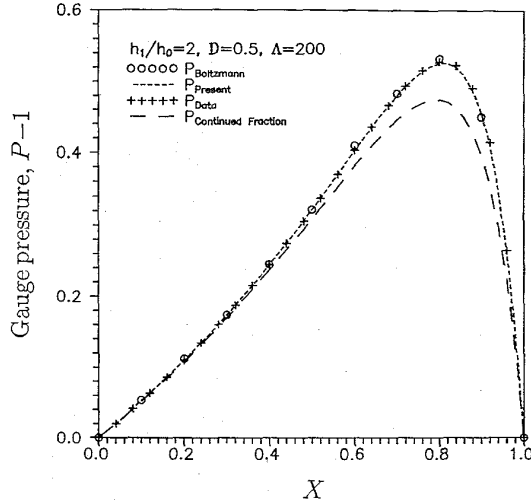


Fig. 3. Comparison of pressure distribution in X-direction resulting from various models.

the rail length); and $X = x/l$ and $Y = y/l$, the normalized coordinates.

NUMERICAL RESULTS AND DISCUSSION

To clarify the new model in (7), the control-volume formulation [9] and the finite difference scheme with the Gauss-Seidel overrelaxation method are utilized in the discretization of the present model. The accuracy of the results is checked by doubling the mesh density, and convergence is achieved when $|(P_{i,j}^{n+1} - P_{i,j}^n)/P_{i,j}^n| < 10^{-10}$ is reached at all nodes.

The gauge pressure, $P-1$, resulting from the modified Reynolds equation, is compared for the infinitely wide slider bearing in Fig. 1. The numerical results in Fig. 3 reveal that P_{Present} and P_{Data} almost accurately approximate $P_{\text{Boltzmann}}$, but $P_{\text{Continued Fraction}}$ underestimates $P_{\text{Boltzmann}}$. In this paper, the normalized load-carrying capacity W is defined as $w/(P_a s l)$, where w is the load-carrying capacity, s is the bearing width (using unit length for infinite width). The result of load W versus D is shown in Fig. 4. It reveals that W_{Present} , W_{Data} , and $W_{\text{Continued Fraction}}$ provide closer approximation to $W_{\text{Boltzmann}}$ than other W , while W_{Data} ($W_{\text{Continued Fraction}}$) is effective only for $D \geq 0.1$ ($8.86 \times 10^{-3} \leq D \leq 8.86 \times 10^{-1}$). On the other hand, W_{Present} shows excellent agreement with $W_{\text{Boltzmann}}$ in the full region of $10^{-4} \leq D \leq 10^2$. This approximation is better than any other model that attempts to approximate W_B in the region of $D \leq 10^{-2}$.

CONCLUSIONS

In this paper, a simple model for describing the ultra-thin film gas lubrication model is proposed. Based on the corrections of higher order terms in the slip-flow velocity boundary conditions, the three optimal adjustable coefficients are determined according to the Boltzmann model.

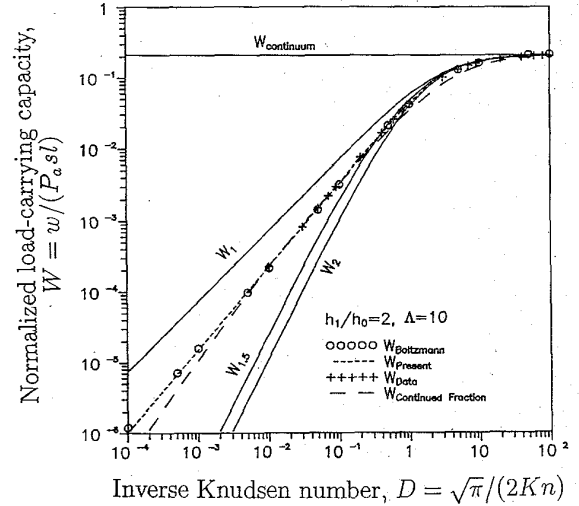


Fig. 4. Comparison of normalized load-carrying capacities versus inverse Knudsen number resulting from various models.

The present model not only reduces the calculation time required, and provides good approximation of pressure flow rate and static load for a wider region of D , it is also useful in approximating the Boltzmann model, and the newly derived equation is widely applicable to practical use. However, the physical interpretations of the three adjustable coefficients remain open.

REFERENCES

- [1] A. Burgdorfer, "The influence of the molecular mean free path on the performance of hydrodynamic gas lubricated bearings," *ASME, J. Basic Eng.*, vol. 81, pp. 94-100, 1959.
- [2] Y.-T. Hsia and G. A. Domoto, "An experimental investigation of molecular rarefaction effects in gas lubricated bearings at ultra-low clearances," *ASME, J. Lub. Tech.*, vol. 105, pp. 120-130, 1983.
- [3] Y. Mitsuya, "Modified Reynolds equation for ultra-thin film gas lubrication using 1.5-order slip-flow model and considering surface accommodation coefficient," *ASME, J. Tribology*, vol. 115, pp. 289-294, 1993.
- [4] R. F. Gans, "Lubrication theory at arbitrary Knudsen number," *ASME, J. Tribology*, vol. 107, pp. 431-433, 1985.
- [5] P. L. Bhatnagar, E. P. Gross, and M. Krook, "A model for collision processes in gases. I. Small amplitude processes in charged and neutral one-component systems," *Phys. Rev.*, vol. 94, pp. 511-525, 1954.
- [6] S. Fukui and R. Kaneko, "Analysis of ultra-thin gas film lubrication based on linearized Boltzmann equation," *ASME, J. Tribology*, vol. 110, pp. 253-261, 1988.
- [7] —, "A database for interpolation of Poiseuille flow rates for high Knudsen number lubrication problems," *ASME, J. Tribology*, vol. 112, pp. 78-83, 1990.
- [8] R. M. Crone, M. S. John, T. E. Karis, and B. Bhushan, "The behavior of a magnetic slider over a rigid-disk surface: A comparison of several approximations of the modified Reynolds equations," *Adv. Inform. Stor. Syst.*, vol. 4, pp. 105-121, 1992.
- [9] Z. Zhao and W. Shi, "A new method for the numerical solution of the Reynolds equation at low spacing," *ASME, J. Tribology*, vol. 115, pp. 83-87, 1993.

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