

Roughness Effects on the Dynamic Coefficients of Ultra-Thin Gas Film in Magnetic Recording

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The stiffness and damping coefficients of a slider gas bearing operating under arbitrary Knudsen number are calculated. A perfect gas is used as the lubricant, and its behavior is described by the modified average Reynolds equation proposed by Makino et al. (1993). The effects of molecular mean free-path on the roughness-induced flow factors are included. The effects of the nondimensional film thickness, H_0 , the surface characteristics, (γ_1, γ_2) , the inverse Knudsen number, D_0 , the nondimensional frequency, Ω , and the modified bearing number, Λ_b , on the dynamic coefficients are discussed in this paper. As expected, the values of dynamic coefficients for various roughness orientations approach the smooth value as the ratio, H_0 , becomes greater and greater, and thus the roughness effect is getting smaller and smaller. The air film of two-sided longitudinal oriented roughness is stiffer than the other two-sided oriented roughnesses is reversed as D_0 greater than some value, and this value is affected significantly by Λ_b . Transversely rough stationary case has the lowest critical value of Λ_b at which the negative translational damping appears. The results show that the roughness effect and rarefaction effect on dynamic coefficients are significant, and they cannot be ignored for stability analysis.

1 Introduction

To increase the recording density in magnetic disk drives, the reduction of spacing between magnetic head and recording medium is of great importance. Under such low spacing (less than $0.1 \mu\text{m}$) the considerations of rarefied gas effect, roughness effect, and dynamic characteristics of the magnetic head become more realistic.

The rarefaction effect is represented by the Knudsen number, K_n , defined as the ratio of the molecular mean free path, λ , to spacing, h . Therefore, in thin-film gas lubrication, the Reynolds equation must be modified by considering the kinetic theory of gases. The modified Reynolds equations based on the velocity boundary of the first-order slip flow (Burgdorfer, 1959), second-order slip flow (Hsia and Domoto, 1983), or 1.5-order slip flow (Mitsuya, 1993) for small Knudsen number, $K_n \ll 1$, have been widely used as lubrication equations. And the static characteristics were compared experimentally (Tanaka et al., 1984; Ohkubo and Kishigami, 1988). The numerical solution of the modified Reynolds equations in the frequency domain (Ono, 1975) and in the time domain (Tang, 1971; Castelli, 1968; White and Nigam, 1980; and Miu and Bogy, 1986b) had been established, and the validity of the dynamic analysis for a conventional flying head slider was reported by comparing the numerical results with experimental measurements (Miu and Bogy, 1986a; Ohkubo et al., 1987; and Tanaka et al., 1989). For arbitrary Knudsen number conditions, analyses based on the Boltzmann equation taking molecular behavior into account were investigated (Gans, 1985; Kubo et al., 1988). The generalized lubrication equation applicable to the ultra-thin spacing of less than $0.1 \mu\text{m}$ ($K_n > 1.6$) was derived (Fukui and Kaneko, 1987, 1988). And the dynamic analyses for an advanced flying

head slider with smooth surface were also reported (Fukui and Kaneko, 1990b; Hayashi et al., 1990).

Numerous research efforts have been devoted to analyzing the surface roughness effects in hydrodynamic lubrication by averaging the film thickness (Michell, 1950; Christensen and Tonder, 1971; Hirs, 1965; Elrod, 1973; Rhow and Elrod, 1974; and Mitsuya, 1984). On the other hand, the average flow model, developed by Patir and Cheng (1978, 1979), has the advantage of being easy to apply to the two-dimensional roughness problem. The average flow rate is expressed by the rate of flow passing through the averaged spacing multiplied by the flow factors. The flow factors are obtained previously by numerical calculation using a small control area exhibiting surface roughness. Elrod (1979) tried to generalize the flow factors analytically. Tripp (1983) outlined the mathematical method using perturbation approach with Green function technique. In recent years, the effects of surface roughness on compressible lubricating films has been studied more and more intensively as the flying head spacing in magnetic storage devices becomes smaller (White, 1980, 1986; Tonder, 1984, 1985; and Green-gard, 1989). Mitsuya et al. (1989) extended the average film concept to a two-dimensional finite slider operated under slip flow regime. Bhushan and Tonder (1989a, 1989b) applied the average Reynolds equation with the roughness flow factors introduced by Tripp (1983) to the two-dimensional finite slider operated under slip flow regime. But they neglected the rarefaction effects on the roughness flow factors. Makino et al. (1993) extended the roughness flow factors for the average Reynolds equation under rarefied gas lubrication. These factors include the local rarefaction effects. The effects of molecular mean free path on the roughness-induced flow factors are included by the assumption that the local compressibility is small. The derivation of flow factors is carried out by means of the perturbation approach with a Green function technique developed by Tripp (1983). The results were presented in terms of average Knudsen number, Peklenik number, and the ratio of mean film thickness

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and standard deviation of composite roughness height. And only the static characteristics were discussed. While the dynamic bearing performance is one of the most important factors in slider operation, analyses have not addressed this aspect of rough slider bearings in detail. The stiffness and damping coefficients are necessary for the dynamic analysis of a rough slider-bearing system. The air bearing characteristics strongly dictate the system's natural frequency (stiffness coefficients) and play a vital role in determining and controlling the peak vibration amplitudes (damping coefficients). Generally, a dynamic bearing system is nonlinear. However, it can be linearized and dynamic coefficients can be predicted for a given set of desired operating conditions.

In this paper, the dynamic characteristics of gas-lubricated slider bearings are analyzed using the modified Reynolds equation proposed by Makino et al. (1993). Using a first-order perturbation method, the time-dependent modified Reynolds equation is transformed in terms of steady-state and dynamic pressures. These equations are solved by a finite difference method. And thus the stiffness and damping coefficients are calculated. The objective of the present paper is to develop an accurate and effective method to calculate both the stiffness and damping coefficients of gas-lubricated slider bearings over a range of bearing numbers, oscillating frequency, average Knudsen number, Peklenik parameter and the ratio of mean

film thickness to the standard deviation of composite roughness height.

2 Analysis

2.1 Basic Equations. For a 2-railed slider flying above a rotating disk (the schematic diagram is shown in Fig. 1(a)), the characteristics of rigid body sliding air bearing with rough surfaces are expressed by linearized bearing coefficients obtained from the perturbed solutions of the modified Reynolds equation considering gas compressibility, rarefaction, and roughness effects. Under the slip flow regime, the modified average Reynolds equation for the finite slider is written in the nondimensional form as (Makino et al., 1993)

$$\frac{\partial}{\partial X} \left(\Gamma_x \cdot \frac{\partial P}{\partial X} \right) + \frac{\partial}{\partial Y} \left(\Gamma_y \cdot \frac{\partial P}{\partial Y} \right) = \frac{\partial}{\partial T} (P \cdot H) + \frac{\partial}{\partial X} [P \cdot (H + H_{s0}^{-1} \phi^S) \cdot u_x] \quad (1)$$

where

$$\Gamma_x = \frac{PH^3}{\Sigma} \cdot (\psi_P \cdot \phi_x^P), \quad \Gamma_y = \frac{PH^3}{\Sigma} \cdot (\psi_P \cdot \phi_y^P) \quad (2)$$

$$\Sigma = 12\mu\omega_0 l^2 / p_a h_0^2 = \text{squeeze number}$$

Nomenclature

a = vibration amplitude of running surface	k_{ij} = air film stiffness ($i = j = 1$: translation mode, $i = j = 2$: pitching mode)	X, Y, Z = nondimensional spatial variable; $x/l, y/l, z/h_0$, respectively
A = nondimensional amplitude of running surface; a/h_0	K_{ij} = nondimensional air film stiffness; $k_{ij} \cdot h_0 / (p_a l^{n+1})$, $n = i + j - 1$	α = accommodation coefficient at boundary surface
b = slider width	K_n = Knudsen number; λ/h	θ_0 = static pitch angle
B = nondimensional slider width; b/l	l = slider length	θ = pitch angle
c_{ij} = air film damping coefficient ($i = j = 1$: translation mode, $i = j = 2$: pitching mode)	m = slider mass	Θ = nondimensional pitch angle; $= \theta/l/h_0$
C_{ij} = nondimensional air film damping coefficient; $c_{ij} \cdot h_0 \omega_0 / (p_a l^{n+1})$; $n = i + j - 1$	M = nondimensional slider mass; $m \cdot \omega_0^2 h_0 / (p_a l^2)$	γ = Peklenik parameter; $\text{ACF}(0.5X)/\text{ACF}(0.5Y)$
D = inverse Knudsen number; $\sqrt{\pi}/(2K_n) = D_0 PH$	p = film pressure	$\Gamma_x = (PH^3/\Sigma) \cdot (\psi_P \cdot \phi_x^P)$
D_0 = inverse Knudsen number defined at minimum spacing h_0	p_a = ambient pressure	$\Gamma_y = (PH^3/\Sigma) \cdot (\psi_P \cdot \phi_y^P)$
$f(D), g(D)$ = coefficient functions of rarefaction	Δp = film pressure variation induced by slider motion	λ_a = molecular mean free path in ambient condition
G_i = complex dynamic pressure coefficients	P = nondimensional film pressure; p/p_a	Λ = bearing number; $l \cdot 6\mu U / (p_a h_0^2)$
h = nominal spacing	ΔP = nondimensional film pressure variation; $= \Delta p/p_a$	Λ_b = modified bearing number; $= B^2 \Lambda$
h_0 = nominal slider trailing edge spacing	t = time	μ = viscosity
h_t = nominal taper height	T = nondimensional time; $\omega_0 t$	σ = standard deviations of composite roughness; $(\sigma_1^2 + \sigma_2^2)^{1/2}$
Δh = spacing fluctuation	u_x = nondimensional surface velocity; $= U/(2l\omega_0) = 1/4\pi$	Σ = squeeze number; $12\mu\omega_0 l^2 / p_a h_0^2$
H = nondimensional spacing; h/h_0	U = surface velocity	Σ_b = modified squeezed number; $B^2 \Sigma$
$H_{s0} = h_0/\sigma$	w = slider static load carrying capacity	ϕ_x^P, ϕ_y^P = pressure flow factors in the x -direction, y -direction
ΔH = nondimensional spacing fluctuation; $\Delta h/h_0$	W = nondimensional load carrying capacity; $w/(p_a l^2)$	ϕ^S = shear flow factor
I = slider moment of inertia about pitch axis	x, y, z = fixed rectangular coordinates	Φ_i^S = single shear flow factor for i th surface
J = nondimensional slider moment of inertia; $I \cdot \omega_0^2 h_0 / (p_a l^4)$	x_G = slider center of gravity	ψ_P = rarefaction coefficient
	X_G = nondimensional center of gravity; $= x_G/l$	ψ = skew angle
	x_G = slider center of gravity	ω = angular frequency;
	x_t = taper length	ω_0 = normalizing angular frequency; $2\pi U/l$
	x_0 = slider pivot point	Ω = nondimensional angular frequency; ω/ω_0
	X_0 = nondimensional pivot point; $= x_0/l$	

$H_{50} = h_0/\sigma$ = the ratio of minimum mean film thickness and the standard deviation of composite roughness height.

$u_x = U/2l\omega_0$ = the nondimensional surface velocity with normalizing angular frequency, $\omega_0 = 2\pi U/l$

And the flow factors derived by Makino et al. (1993) are as follows:

$$\phi_x^p = 1 + g \cdot \left(\frac{\sigma}{h}\right)^2 \cdot \left(1 - \frac{f^2}{g} \cdot \frac{1}{\gamma + 1}\right),$$

$$\phi_y^p = 1 + g \cdot \left(\frac{\sigma}{h}\right)^2 \cdot \left(1 - \frac{f^2}{g} \cdot \frac{\gamma}{\gamma + 1}\right) \quad (3)$$

$$\phi^s = \left(\frac{\sigma_1}{\sigma}\right)^2 \cdot \Phi_1^s - \left(\frac{\sigma_2}{\sigma}\right)^2 \cdot \Phi_2^s \quad (4)$$

where ϕ_x^p and ϕ_y^p are the pressure flow factors in the x - and y -direction, and Φ_i^s is the single shear flow factor for i th surface defined as

$$\Phi_i^s = f \cdot \left(\frac{\sigma}{h}\right) \cdot \frac{1}{\gamma_i + 1} \quad (5)$$

and

σ = standard deviation of composite roughness height.
 γ = Peklenik parameter (Peklenik, 1967)

The coefficient functions of rarefaction f , g (Makino et al., 1993) are found to be

$$f(D) = \left\{ \sum_{k=0}^3 [(3-k) \cdot C_k \cdot D^{-k}] \right\} / \psi_p \quad (6)$$

$$g(D) = (3 \cdot C_0 + C_1 \cdot D^{-1}) / \psi_p \quad (7)$$

where the rarefaction coefficient ψ_p valid for arbitrary Knudsen numbers (proposed by Fukui and Kaneko, 1990a) is defined as

$$\begin{aligned} \frac{\partial}{\partial X} & \left\{ \frac{1}{\Sigma} \left[(\psi_p)_0 \cdot (\phi_x^p)_0 \cdot (P_0 \cdot 3H_0^2 \Delta H + H_0^3 \Delta P) \cdot \frac{\partial P_0}{\partial X} + P_0 \cdot H_0^3 \cdot (\psi_p)_0 \cdot \left(\frac{\partial \phi_x^p}{\partial P} \Delta P + \frac{\partial \phi_x^p}{\partial H} \Delta H \right)_0 \cdot \frac{\partial P_0}{\partial X} \right] \right. \\ & \left. + P_0 \cdot H_0^3 \cdot (\phi_x^p)_0 \cdot \left(\frac{\partial \psi_p}{\partial P} \Delta P + \frac{\partial \psi_p}{\partial H} \Delta H \right)_0 \cdot \frac{\partial P_0}{\partial X} + (P_0 \cdot H_0^3 \cdot (\psi_p)_0 \cdot (\phi_x^p)_0) \cdot \frac{\partial \Delta P}{\partial X} \right\} \\ & + \frac{\partial}{\partial Y} \left\{ \frac{1}{\Sigma} \left[(\psi_p)_0 \cdot (\phi_y^p)_0 \cdot (P_0 \cdot 3H_0^2 \Delta H + H_0^3 \Delta P) \cdot \frac{\partial P_0}{\partial Y} + P_0 \cdot H_0^3 \cdot (\psi_p)_0 \cdot \left(\frac{\partial \phi_y^p}{\partial P} \Delta P + \frac{\partial \phi_y^p}{\partial H} \Delta H \right)_0 \cdot \frac{\partial P_0}{\partial Y} \right] \right. \\ & \left. + P_0 \cdot H_0^3 \cdot (\phi_y^p)_0 \cdot \left(\frac{\partial \psi_p}{\partial P} \Delta P + \frac{\partial \psi_p}{\partial H} \Delta H \right)_0 \cdot \frac{\partial P_0}{\partial Y} + (P_0 \cdot H_0^3 \cdot (\psi_p)_0 \cdot (\phi_y^p)_0) \cdot \frac{\partial \Delta P}{\partial Y} \right\} \\ & = \frac{\partial}{\partial X} \left[(P_0 \cdot \Delta H + H_0 \cdot \Delta P) \cdot u_x + H_{50}^{-1} \cdot u_x \cdot P_0 \cdot \left(\frac{\partial \phi^s}{\partial P} \Delta P + \frac{\partial \phi^s}{\partial H} \Delta H \right)_0 + H_{50}^{-1} \cdot u_x \cdot (\phi^s)_0 \cdot \Delta P \right] \\ & + \frac{\partial}{\partial T} [P_0 \cdot \Delta H + H_0 \cdot \Delta P] \quad (16) \end{aligned}$$

$$\psi_p = \sum_{k=0}^3 C_k \cdot D^{-k} \quad (8)$$

where D is the inverse Knudsen number $= \sqrt{\pi}/2K_n$.

From the above, two important roughness factors are introduced: the mean film thickness-roughness ratio, H_s , which shows the relative importance of roughness on bearing performance; and the surface pattern parameter, γ , which is visualized as the length-to-width ratio of a representative asperity.

2.2 Perturbation Method. Assuming that the slider vibrates about its mean steady-state position given by Z_0, Θ_0 for the first-order perturbation, the pressure and film thickness can be expressed as

$$P = P_0 + \Delta P \quad (9)$$

and

$$H = H_0 + \Delta H \quad (10)$$

where P_0 and H_0 are the static pressure and film thickness, and ΔP and ΔH are the dynamic pressure and film thickness.

The flow factors and the rarefaction coefficient may thus be expressed as

$$\phi_x^p = (\phi_x^p)_0 + \left(\frac{\partial \phi_x^p}{\partial P} \right)_0 \cdot \Delta P + \left(\frac{\partial \phi_x^p}{\partial H} \right)_0 \cdot \Delta H \quad (11)$$

$$\phi_y^p = (\phi_y^p)_0 + \left(\frac{\partial \phi_y^p}{\partial P} \right)_0 \cdot \Delta P + \left(\frac{\partial \phi_y^p}{\partial H} \right)_0 \cdot \Delta H \quad (12)$$

$$\phi^s = (\phi^s)_0 + \left(\frac{\partial \phi^s}{\partial P} \right)_0 \cdot \Delta P + \left(\frac{\partial \phi^s}{\partial H} \right)_0 \cdot \Delta H \quad (13)$$

$$\psi_p = (\psi_p)_0 + \left(\frac{\partial \psi_p}{\partial P} \right)_0 \cdot \Delta P + \left(\frac{\partial \psi_p}{\partial H} \right)_0 \cdot \Delta H \quad (14)$$

Substituting Eqs. (9)–(14) into Eq. (1) and neglecting the higher-order terms, it yields the zeroth-order equation:

$$\begin{aligned} \frac{\partial}{\partial X} & \left[\frac{P_0 H_0^3}{\Sigma} (\psi_p)_0 \cdot (\phi_x^p)_0 \cdot \frac{\partial P_0}{\partial X} \right] \\ & + \frac{\partial}{\partial Y} \left[\frac{P_0 H_0^3}{\Sigma} (\psi_p)_0 \cdot (\phi_y^p)_0 \cdot \frac{\partial P_0}{\partial Y} \right] \\ & = \frac{\partial}{\partial X} [(H_0 + (\phi_x^p)_0 \cdot H_{50}^{-1}) \cdot u_x \cdot P_0] \quad (15) \end{aligned}$$

and the first-order equations:

The nondimensional boundary conditions are

$$P = 1 \quad \text{at } X = 0, \quad \text{and } X = 1$$

$$P = 1 \quad \text{at } Y = 0, \quad \text{and } Y = B$$

$$\Delta P = 0 \quad \text{at } X = 0, \quad \text{and } X = 1$$

$$\Delta P = 0 \quad \text{at } Y = 0, \quad \text{and } Y = B \quad (17)$$

To simplify the calculations, this study assumes a model with

freedom of movement in the translation and pitching modes. In this "two-degree-of-freedom" model, the bearing coefficients are expressed by a 2×2 matrix. Assuming a sinusoidal disturbance and response of the slider, spacing fluctuation ΔH is expressed as

$$\Delta H = Z + \Theta \cdot (X_G - X) - Z_d \quad (18)$$

where

Z = translational displacement at the slider center of gravity
 Θ = pitch angle about the steady state angle Θ_0
 $Z_d = A \cdot \sin(\Omega T)$ = translational displacement of disk harmonic vibration
 A = nondimensional vibration amplitude (a/h_0)

Dynamic pressure ΔP for spacing fluctuation ΔH in Eq. (18) is expressed as

$$\Delta P = G_1 \cdot Z + G_2 \cdot \Theta + G_3 \cdot A \cdot \sin(\Omega T) \quad (19)$$

where G_i are complex quantities dependent on position X and Y .

Then, taking the Laplace transformation with respect to non-dimensional time, $T = \omega_0 t$, we can obtain the differential equations governing the complex quantities G_i ($i = 1, 2$ or 3).

$$\begin{aligned} \frac{\partial}{\partial X} \left\{ \frac{1}{\Sigma} \left[(\psi_r)_0 \cdot (\phi_x^r)_0 \cdot (P_0 \cdot 3H_0^2 F_i + H_0^3 G_i) \cdot \frac{\partial P_0}{\partial X} + P_0 \cdot H_0^3 \cdot (\psi_r)_0 \cdot \left(\frac{\partial \phi_x^r}{\partial P} G_i + \frac{\partial \phi_x^r}{\partial H} F_i \right)_0 \cdot \frac{\partial P_0}{\partial X} \right] \right. \\ \left. + P_0 \cdot H_0^3 \cdot (\phi_x^r)_0 \cdot \left(\frac{\partial \psi_r}{\partial P} \cdot G_i + \frac{\partial \psi_r}{\partial H} \cdot F_i \right)_0 \cdot \frac{\partial P_0}{\partial X} + (P_0 \cdot H_0^3 \cdot (\psi_r)_0 \cdot (\phi_x^r)_0) \cdot \frac{\partial G_i}{\partial X} \right\} \\ + \frac{\partial}{\partial Y} \left\{ \frac{1}{\Sigma} \left[(\psi_r)_0 \cdot (\phi_y^r)_0 \cdot (P_0 \cdot 3H_0^2 F_i + H_0^3 G_i) \cdot \frac{\partial P_0}{\partial Y} + P_0 \cdot H_0^3 \cdot (\psi_r)_0 \cdot \left(\frac{\partial \phi_y^r}{\partial P} G_i + \frac{\partial \phi_y^r}{\partial H} F_i \right)_0 \cdot \frac{\partial P_0}{\partial Y} \right] \right. \\ \left. + P_0 \cdot H_0^3 \cdot (\phi_y^r)_0 \cdot \left(\frac{\partial \psi_r}{\partial P} \cdot G_i + \frac{\partial \psi_r}{\partial H} \cdot F_i \right)_0 \cdot \frac{\partial P_0}{\partial Y} + (P_0 \cdot H_0^3 \cdot (\psi_r)_0 \cdot (\phi_y^r)_0) \cdot \frac{\partial G_i}{\partial Y} \right\} \\ = \frac{\partial}{\partial X} \left[(P_0 \cdot F_i + H_0 \cdot G_i) \cdot u_x + H_{s0}^{-1} \cdot u_x \cdot P_0 \cdot \left(\frac{\partial \phi^s}{\partial P} \cdot G_i + \frac{\partial \phi^s}{\partial H} \cdot F_i \right)_0 + H_{s0}^{-1} \cdot u_x \cdot (\phi^s)_0 \cdot G_i \right] \\ + j\Omega \cdot [P_0 \cdot F_i + H_0 \cdot G_i] \quad (20) \end{aligned}$$

where

$$F_1 = 1, F_2 = X_G - X, F_3 = -1, \text{ and } j = \sqrt{-1}.$$

and the boundary conditions are

$$\begin{aligned} G_i &= 0 \text{ at } X = 0, \text{ and } X = 1 \\ G_i &= 0 \text{ at } Y = 0, \text{ and } Y = B \end{aligned} \quad (21)$$

2.3 Modeling of the Gas Film. The dynamic film force and moment about pivot point can be modeled as (as shown in Fig. 1(b), Hayashi et al., 1990)

$$\begin{aligned} - \int_0^1 \int_0^B \Delta P dY dX &= C_{11} \frac{dZ}{dT} + C_{12} \frac{d\Theta}{dT} \\ &+ K_{11} Z + K_{12} \Theta - \int_0^1 \int_0^B G_3 A \sin(\Omega T) dY dX \quad (22) \\ - \int_0^1 \int_0^B \Delta P (X_G - X) dY dX \\ &= C_{21} \frac{dZ}{dT} + C_{22} \frac{d\Theta}{dT} + K_{21} Z + K_{22} \Theta \end{aligned}$$

$$- \int_0^1 \int_0^B G_3 (X_G - X) A \sin(\Omega T) dY dX \quad (23)$$

From Eqs. (19), (22), and (23), we find

$$\begin{aligned} K_{11} + j\Omega C_{11} &= - \int_0^1 \int_0^B G_1 dY dX \\ K_{12} + j\Omega C_{12} &= - \int_0^1 \int_0^B G_2 dY dX \\ K_{21} + j\Omega C_{21} &= - \int_0^1 \int_0^B G_1 (X_G - X) dY dX \\ K_{22} + j\Omega C_{22} &= - \int_0^1 \int_0^B G_2 (X_G - X) dY dX \quad (24) \end{aligned}$$

2.4 Analysis Procedure. The control-volume formulation is utilized in the discretization of the steady modified Reynolds Eq. (15), and the dynamic equations (20). For the solution of elliptic partial differential equations, Smith (1971) showed that the Gauss-Seidel over-relaxation iterative method was the most efficient. This method is then used to solve the finite difference equations. After the static pressure distribution is obtained, substitute the pressure distribution into Eqs. (20), and the dynamic complex quantities are obtained by solving the

discretized dynamic equations. And thus, the dynamic coefficients are obtained from the dynamic complex quantities together with the Eqs. (24).

3 Results and Discussion

The finite difference scheme, with Gauss-Seidel over-relaxation method, is applied to solve Eq. (15) and Eqs. (20) numerically. The accuracy of results is checked by doubling the mesh density. Errors are less than one percent. Convergence is achieved until 10^{-5} of $|(G_{ij}^{n+1} - G_{ij}^n)/G_{ij}^n|$ is reached at all nodes. In order to confirm the accuracy and reliability of this program, the results are in good agreement with the static analysis results proposed by Makino et al. (1993), (as shown in Fig. 2(a)) and the dynamic coefficients (smooth case) proposed by Hayashi et al. (1990), (as shown in Fig. 2(b)). Typical operation conditions based on fixed configuration are as follow:

Slider length $l = 0.5$ mm
 Slider width $b = 0.0515$ mm
 Taper length $x_t = 0.05$ mm
 Taper height $h_t = 0.4356$ μ m
 Min. film thickness $h_0 = 0.05$ μ m
 Pitch angle $\theta_0 = 0.000202$
 Modified bearing number $\Lambda_p = 12$
 the inverse Knudsen number $D_0 = 0.69$

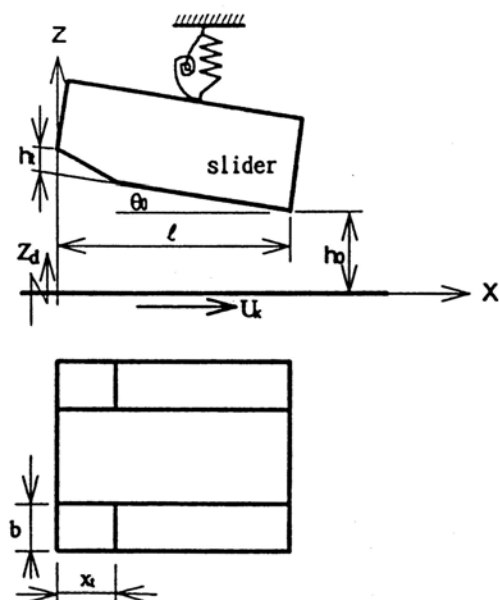


Fig. 1(a) Slider geometry

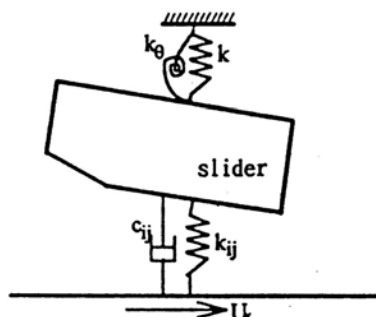


Fig. 1(b) Gas film model

Standard deviation of composite roughness $\sigma = 0.025 \mu\text{m}$
 Static load $w = 0.307 \text{ mN}$
 Slider Mass $m = 0.3 \text{ mg}$
 Moment of inertia $I = 0.0075 \text{ mg} \cdot \text{mm}^2$
 Lubricant: air in standard atmospheric conditions.
 Accommodation coefficients of boundary surface: $\alpha = 1$
 (assume diffuse reflection)

The two-sided roughness effects on non dimensional air film dynamic coefficients for various roughness orientations, K_{ij} and C_{ij} ($i, j = 1, 2$), as shown in Fig. 3(a)–Fig. 3(h) are found from Eqs. (15), (20), and (24). For two-sided roughness cases, both surfaces have the same roughness properties, the shear flow factor becomes zero. As expected, the values of dynamic coefficients for various roughness orientations approach the smooth value as the ratio, $H_{s0} = h_0/\sigma$, becomes greater and greater, and the roughness effect is getting smaller and smaller. For longitudinal ($\gamma = 9$) and isotropic ($\gamma = 1$) oriented roughness cases, the air film is stiffer in low H_{s0} region and is more compliant in high H_{s0} region. On the contrary, transversely ($\gamma = \frac{1}{9}$) oriented roughness case has the reverse tendency.

In Fig. 3(a), longitudinal oriented roughness ($\gamma = 9$) has the largest translational stiffness value (K_{11}) and transversely oriented roughness ($\gamma = \frac{1}{9}$) has the lowest translational stiffness value. For isotropic oriented roughness ($\gamma = 1$), the translational stiffness value is larger than that of smooth case. As pointed out by Hayashi et al. (1990), the translational stiffness K_{11} depends almost uniquely on load capacity. This variation can be explained directly by the variation of load capacity for each roughness orientation (Makino, et al., 1993). For this slender slider, the width to length ratio (b/l) is 0.103, the pres-

sure generation is dominated by pressure flow in the y -direction, namely, the side leakage flow. The restrictions of side leakage flow by longitudinal roughness is greater than that of the smooth case. Though the isotropic roughness still restricts the side flow, the load capacity of isotropic roughness is slightly greater than the smooth value. On the contrary, the transverse roughness enhance the side flow, so we have the lowest load capacity for transverse case and the largest load capacity for longitudinal case.

Figure 3(b) and (c) show the cross stiffnesses K_{12} and K_{21} as a function of the ratio $H_{s0} (=h_0/\sigma)$ for various roughness orientations. The values of K_{12} and K_{21} are all negative, and the variation and tendency of K_{12} and K_{21} for various oriented roughnesses are the same. The negative sign means that the increment in dynamic moment (or dynamic force) introduces reduction in dynamic displacement (or dynamic angular displacement). Figure 3(d) is the pitching stiffness K_{22} as function of the ratio H_{s0} for various roughness orientations. From Figs. 3(a)–3(d), the absolute value of stiffnesses have the same behavior: $(\gamma = 9) > (\gamma = 1) > (\text{smooth}) > (\gamma = \frac{1}{9})$. That is, the air film for the longitudinal oriented case is the stiffest and that for transversely oriented roughness case is the most compliant. For damping coefficients, Figs. 3(e)–3(h), the orders for various roughness orientations and smooth case are reversed as that for the corresponding stiffness mode (i.e., C_{12} has the same sequence for various roughness orientations as that of K_{12}) except C_{11} . But the order of the results for various roughness orientation cases and smooth case has no exception on C_{11} for D_0 values greater than 0.7 (Fig. 4(a)). In Fig. 3(e), the translational damping coefficients C_{11} for longitudinal roughness orientation ($\gamma = 9$) at H_{s0} below 3.0 has the reverse

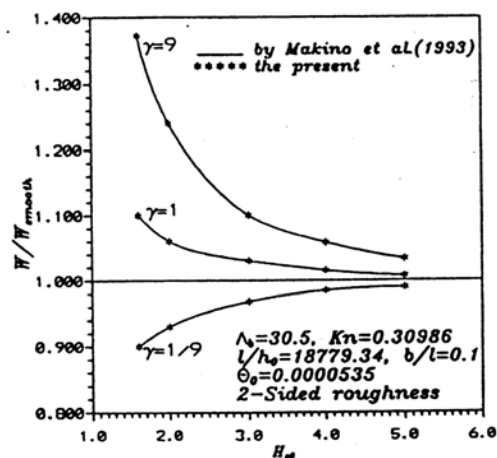


Fig. 2(a) Comparison of load ratio with Makino et al. (1993)

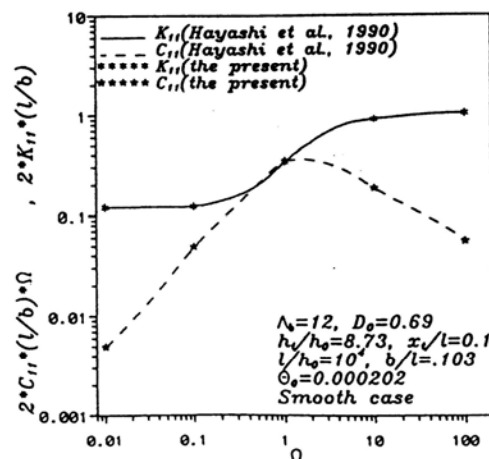


Fig. 2(b) Comparison of dynamic coefficient with Hayashi et al. (1990)

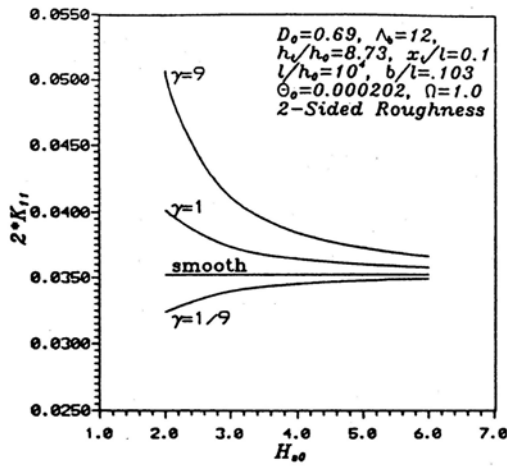


Fig. 3(a) Two-sided roughness effects on air film stiffness coefficients K_{11} for various roughness orientations

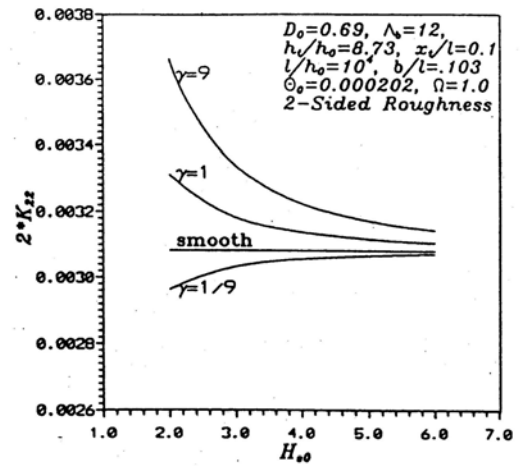


Fig. 3(d) Two-sided roughness effects on air film stiffness coefficients K_{22} for various roughness orientations

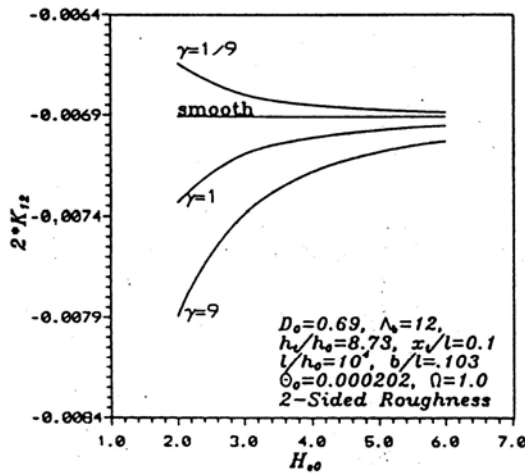


Fig. 3(b) Two-sided roughness effects on air film stiffness coefficients K_{12} for various roughness orientations

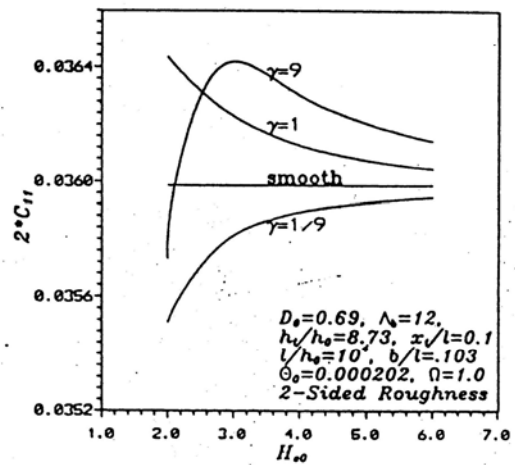


Fig. 3(e) Two-sided roughness effects on air film damping coefficients C_{11} for various roughness orientations

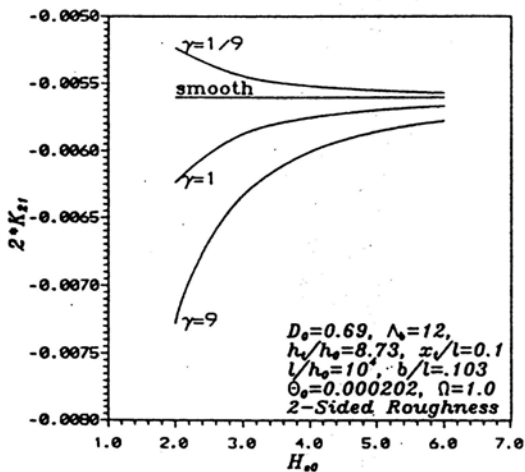


Fig. 3(c) Two-sided roughness effects on air film stiffness coefficients K_{21} for various roughness orientations

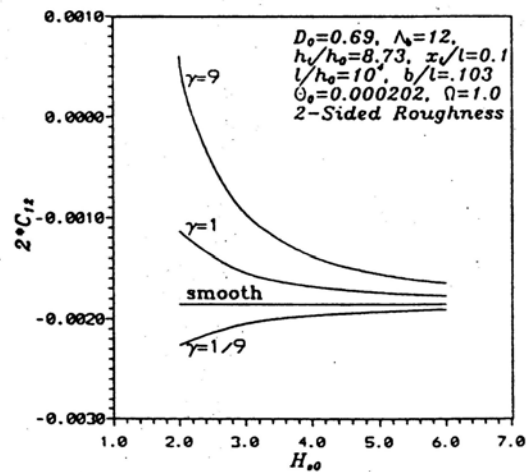


Fig. 3(f) Two-sided roughness effects on air film damping coefficients C_{12} for various roughness orientations

tendency as that for the other orientation, i.e., it departs from the smooth value as H_{s0} grows. This can be seen in Fig. 4(a) that the "reversed region" for D_0 on C_{11} is about 0.6–0.7. In Fig. 3(e), D_0 (=0.69) is falling in the region. The reversed region means that the effect on C_{11} for various roughness orientations are reversed as the value of D_0 are falling on different sides outside this reversed region. So the exception on C_{11} does not exist for D_0 greater than 0.7.

Figure 4(a) shows K_{11} and C_{11} as a function of the inverse Knudsen number D_0 , defined by the minimum spacing, with Λ_b

= 12. As D_0 increases, the rarefied gas effect is insignificant, so K_{11} increases, and the values for each roughness orientation approach the continuum value of K_{11} . This is because the air film contains more molecules at higher D_0 condition than that at lower D_0 condition. When the value of D_0 is smaller than 0.6, C_{11} increases as D_0 increases. When the value of D_0 is larger than 0.7, the effect for different roughness orientations are reversed, C_{11} decreases as D_0 increases, and the discrepancy become large. That is, the rarefied effects, always diminish the roughness effects on C_{11} . In Fig. 4(b) (Λ_b = 120), the reversed

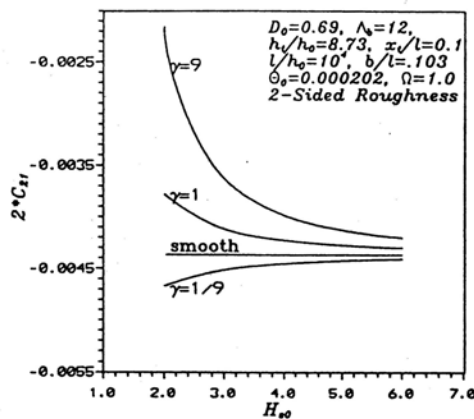


Fig. 3(g) Two-sided roughness effects on air film damping coefficients C_{21} for various roughness orientations

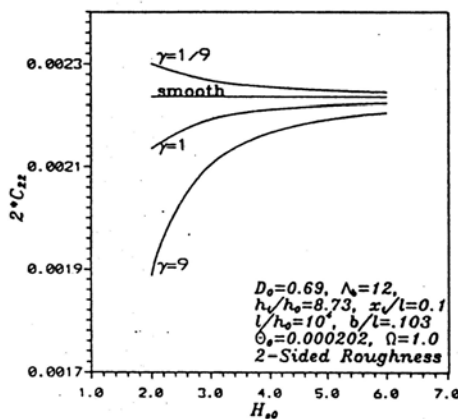


Fig. 3(h) Two-sided roughness effects on air film damping coefficients C_{22} for various roughness orientations

region for D_0 on C_{11} is about 0.06–0.065. The reversed region for D_0 on C_{11} is influenced by the modified bearing number.

Figure 5(a) shows K_{11} and C_{11} , as a function of nondimensional frequency Ω for various roughness orientations, with $\Lambda_b = 150$. It is noteworthy that the dynamic coefficients are nearly constant at low frequency, Ω . At high frequencies, the lubricant compressibility is significant, and the dynamic coefficients are dependent on the frequency of vibrations (Goodwin, 1989). The lubricant appears to be compressed as well as to be squeezed out of the clearance space as the two surfaces move toward each other. If the two surfaces move toward one another with very low relative velocities, this operation may be thought as the steady-state case and the compressibility effect will be less significant. For two-sided longitudinal roughness orientation ($\gamma = 9$), translational damping coefficient C_{11} becomes negative in the frequency range of $\Omega = 0.22$ – 0.72 for $\Lambda_b = 150$. As pointed out by Hayashi et al. (1990) for smooth case, translational damping coefficient decreases as Λ_b increases and they become negative in the frequency range of $\Omega = 0.4$ – 0.6 for $\Lambda_b = 180$. The effect of modified bearing number Λ_b on air film damping coefficients with $\Omega = 0.5$ and $\gamma = 9$ for two-sided roughness, smooth stationary, rough stationary, and smooth cases with longitudinal roughness orientation ($\gamma = 9$) are shown in Fig. 5(b). The translational damping coefficients become negative for Λ_b greater than 120 except in the smooth case. The rough stationary case, $(\gamma_1, \gamma_2) = (0, 9)$, has the lowest critical value of Λ_b at which the negative damping appears. From the above, the roughness effects have a significant effect on the negative damping initiation, and will be the mechanism of system instability (Fukui and Kaneko, 1990b; Ono, 1975).

K_{11} and C_{11} of this taper-flat slider as a function of the ratio H_{s0} ($=h_0/\sigma$) for two-sided roughness, smooth stationary, rough stationary and smooth cases are shown in Figs. 6(a) and (b), respectively. As H_{s0} increases, K_{11} and C_{11} approach the smooth value for each case. In Fig. 6(a), the sequence of the magnitude of K_{11} for these three cases are: (rough stationary case) > (two-sided roughness case) > (smooth stationary case). For C_{11} values, the sequence of two-sided roughness, smooth stationary and rough stationary cases is reversed from that for K_{11} . The variation of K_{11} or C_{11} between these three cases for transverse roughness ($\gamma = \frac{1}{9}$) is significant, but that for longitudinal roughness ($\gamma = 9$) is slight. For rough stationary case, K_{11} is greater for various orientations than the smooth value.

Due to the complex properties of the gas film, the dynamic coefficients are influenced by many parameters, i.e., slider geometry, operation conditions, non dimensional frequency, roughness parameters (H_{s0} and γ) and rarefied gas effects, etc. This paper discusses the tendency and variation of the dynamic coefficients numerically for one parameter with other parameters fixed. Further analysis of stability is needed in the future.

4 Conclusion

This paper presents the numerical calculation of the dynamic coefficients of gas-lubricated slider bearings under arbitrary Knudsen number conditions using a modified lubrication equation based on average flow model for roughness effect, linearized Boltzmann equation for rarefaction effect and considering the rarefaction effect on roughness flow factors. A perturbation

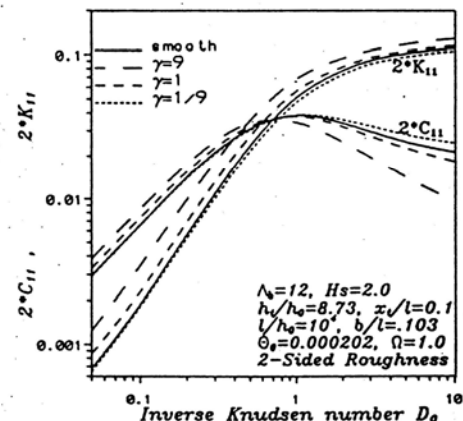


Fig. 4(a) Influence of inverse Knudsen number D_0 on air film stiffness and damping coefficients, K_{11} and C_{11} , for various roughness orientations

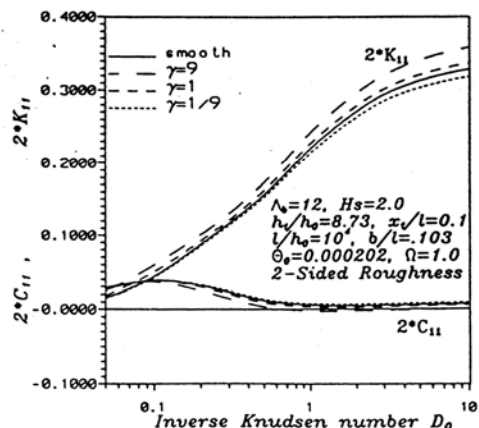


Fig. 4(b) Influence of inverse Knudsen number D_0 on air film stiffness and damping coefficients, K_{11} and C_{11} , for various roughness orientations

method is used to obtain the linearized dynamic coefficient equations, and a finite difference scheme with over-relaxation method is used to solve the eight dynamic coefficients. The results show that

1. The influence of roughness effects (H_{s0} and γ) and the inverse Knudsen number D_0 on the dynamic coefficients are significant and it cannot be ignored in stability analysis.
2. The values of dynamic coefficients for various roughness orientations approach the smooth value as the ratio $H_{s0} = h_0/\sigma$ increases, and then the roughness effect becomes insignificant.
3. For two-sided longitudinal oriented roughness ($\gamma = 9$), the air film is stiffer than that of the other oriented ($\gamma = 1$, $\gamma = \frac{1}{9}$ and smooth) cases studied.
4. For the translational damping coefficients for D_0 greater than 0.7, the effects for various two-sided roughness orientations and the smooth case are reversed compared to that for the corresponding stiffness mode, i.e., ($\gamma = 9$) < ($\gamma = 1$) < (smooth) < ($\gamma = \frac{1}{9}$). There exists a reversed region for the damping coefficients to reverse their variation for various roughness orientations. And it differs for different modified bearing numbers.
5. The influence of frequency on the dynamic coefficients is very small at lower frequencies, but the influence is significant at higher frequencies.
6. Negative damping can occur at smaller modified bearing numbers for two-sided longitudinal oriented roughness than for the smooth case. The transversely rough stationary case, (γ_1 ,

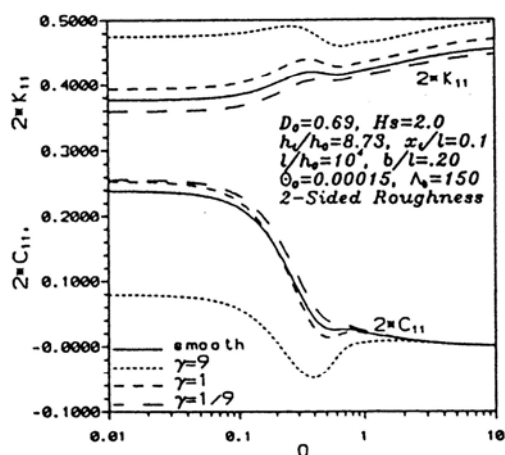


Fig. 5(a) Relationship between nondimensional frequency Ω on air film stiffness and damping coefficients, K_{11} and C_{11} , for various roughness orientations

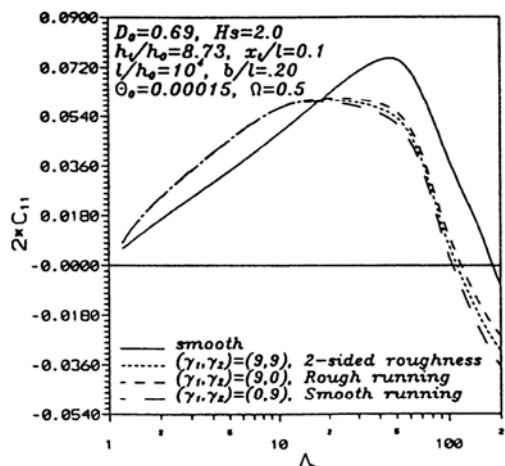


Fig. 5(b) Effect of modified bearing number Λ_0 on air film damping coefficients C_{11} , for roughness stationary, smooth stationary, two-sided roughness and smooth cases

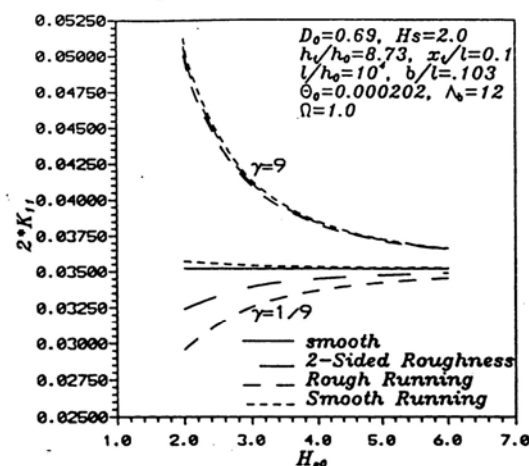


Fig. 6(a) Comparison of smooth stationary, rough stationary, two-sided roughness and smooth cases effects on air film stiffness coefficients K_{11} , for various roughness orientations

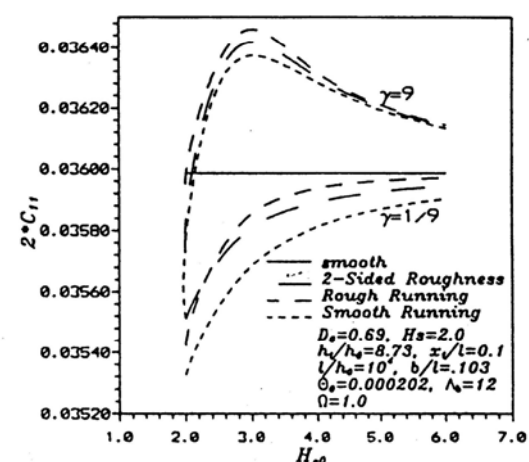


Fig. 6(b) Comparison of smooth stationary, rough stationary, two-sided roughness and smooth cases effects on air film damping coefficients C_{11} , for various roughness orientations

γ_2) = (0, 9), has the lowest critical value of Λ_b at which the negative damping appears. We cannot say which one is more stable unless a stability analysis of this system is carried out.

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