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Pergamon

*Int. J. Heat Mass Transfer.* Vol. 40, No. 7, pp. 1716–1719, 1997
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0017 9310/97 $17.00+0.00

PII: S0017–9310(96)00220–7

# The unsteady solutions of a unified heat conduction equation

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(*Received* 8 *January* 1996)

## 1. INTRODUCTION

The short-phase laser heating process of metals is composed of three general steps: the deposition of radiation energy on electrons, the transport of energy by electrons and the propagation of energy through media. The propagation of energy during a relatively slow heating process can be modeled by the Fourier heat conduction model, since the deposition of radiation energy can be assumed to be instantaneous. However, it takes time, in reality, to establish an equilibrium state in thermodynamic transition. For a problem involving reflectivity change resulting from short-pulse laser heating on gold films [1], the response time is on the order of picoseconds, comparable to the time required to establish an equilibrium state. The diffusion theory fails under such circumstances because the hot electron gas and the metal lattice cannot reach thermodynamic equilibrium in such a short period of time. Thus, more general and rigorous models are needed to include effects of electron-lattice interactions and non-Fourier transport. After Maxwell's research [2] on the kinetic theory of gases, which has had great influence on the development of the thermal wave theory, modifications on Fourier's law are promoted by its deficiencies in advanced applications [3–16].

The unified model (Tzou [16]) is a generalized approach based on the dual-phase-lag concept which accounts for the lagging behavior in the high-rate response. A universal constitutive equation between the heat flux vector and the temperature gradient is proposed with an effort to cover a wide range of physical responses from microscopic to macroscopic scales in both space and time [16]. An exact solution, using the method of separation of variables, to the above universal constitutive equation for a one-dimensional problem is addressed in this paper. Part of the results are found to be different from those by Tzou [16]. The aim of this note is to present a convenient approach to the short-pulse laser heating problem by virtue of the unified heat conduction equation.

## 2. THEORETICAL ANALYSIS

The short-pulse laser heating of a metal film can be treated as a one-dimensional problem because its heat penetration depth is much smaller than the beam diameter. The solid is assumed to have a finite thickness, $l$. The phase lag of the heat flux and that of the temperature gradient are $\tau_q$ and $\tau_T$, respectively. An initial temperature distribution of constant value, $T_0$, in solid and an imposed initial time-rate change of temperature, $\dot{T}_0$, are given. A suddenly-raised temperature $T_w$ at left end $x \times 0$ and a zero temperature gradient remaining at right end $x = l$ are suitable boundary conditions for this type of problem. After introducing the following dimensionless variables as in ref. [16],

$$\theta = \frac{T - T_0}{T_w - T_0}, \quad \delta = \frac{x}{l}, \quad \text{and} \quad \beta = \frac{t}{(l^2/\alpha)} \tag{1}$$

the temperature field equation, the initial conditions and the boundary conditions become:

$$\frac{\partial^2\theta}{\partial\delta^2} + z_T \frac{\partial^3\theta}{\partial\delta^2\partial\beta} = \frac{\partial\theta}{\partial\beta} + z_q \frac{\partial^2\theta}{\partial\beta^2} \tag{2}$$

and

$$\theta = 0 \quad \text{and} \quad \frac{\partial\theta}{\partial\beta} = \dot{\theta}_0 \quad \text{at } \beta = 0 \tag{3}$$

$$\theta = 1 \quad \text{at } \delta = 0 \quad \text{and} \quad \frac{\partial\theta}{\partial\delta} = 0 \quad \text{at } \delta = 1 \tag{4}$$

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