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The nonlinear Czochralski growing process of crystals with variable thermal properties

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Abstract—The nonlinear dynamic process of Czochralski crystal growth with variable thermal properties is studied by the perturbation method. After evaluating the order of magnitude of these thermal properties, it is found that the temperature-dependent thermal conductivity is the dominant parameter in this problem. Comparing the results with that of the constant thermal conductivity model, we find that at the initial transient stage the difference between these models is insignificant, while the model difference is enlarged gradually, and at the final steady stage the model difference is the largest. Moreover, during the process, the higher the temperature difference between the crystal melting point and the surroundings is, the larger is the model difference. Finally, to avoid the computational burden, in this article, an optimal constant value of thermal conductivity is recommended. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Recently, Hwang *et al.* [1] have studied the nonlinear dynamic process of Czochralski (CZ) crystal growth with constant radius by the perturbation method and found that for growth under constant bulk melt temperature (pull rate), the pull rate (bulk melt temperature) would increase with increasing ambient Biot number for convective heat transfer from melt. Also, it was shown that the pull rate must not exceed the maximum value, otherwise the process will break down if the melt could not be supercooled (cf. ref. [1]).

However, conclusions based on the theoretical description of any physical process are strongly dependent on a knowledge of the numerical values of the various parameters appearing in the formalism. Hence, it is imperative to critically survey the available information concerning the parameters relevant to a realistic modeling and analysis of CZ crystal growth [2]. When temperature variations are large or the thermal properties vary rapidly with temperature, a valid description of the heat-conduction problem must take into account the variation of the transport coefficients with temperature. Then the differential equation of energy transport becomes nonlinear. Because the nonlinearity of energy balance equation and the time-dependent boundary conditions in CZ process eliminate the possibility of finding an exact analytical solution, the finite-element (or difference) solutions of the nonlinear CZ crystal growth model were used to predict crystal temperature distributions, pull rate and the thermal stresses by many authors [3–9]. Hwang *et al.* [10, 11] successfully adopted the perturbation method to investigate the effects of wall conduction and interface thermal resistance on the phase-change problem, as well as analysed unsteady thermoelastic dislocation generation during Czochralski crystal growth. Aziz and Lunardini [12] reviewed perturbation techniques in phase change heat transfer. The variable-property Stefan problem in which the thermal conductivity and specific heat of the solid phase are temperature-dependent has been studied by Aziz [13] and Pedroso and Domoto [14] using a regular perturbation method.

In the present work, the perturbation technique is extended to the nonlinear dynamic model of CZ crystal growth to examine the effect of variable thermal properties on the process. A comparison is made between the present results and the results of the temperature-independent thermal properties model to show the effect of variable thermal properties on the temperature distributions and the pull rate of the crystal as well as the temperature of the bulk melt during the process.

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