

Electroosmotic Entry Flow in a Microchannel

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The entry flow induced by an applied electrical potential through microchannels between two parallel plates is analyzed in this work. A nonlinear, two-dimensional Poisson equation governing the applied electrical potential and the zeta potential of the solid–liquid boundary and the Nernst–Planck equation governing the ionic concentration distribution are numerically solved using a finite-difference method. The applied electrical potential and zeta potential are unified in the Poisson equation without using linear superposition. A body force caused by the interaction between the charge density and the applied electrical potential field is included in the full Navier–Stokes equations. The effects of the entrance region on the fluid velocity distribution, charge density boundary layer, entrance length, and shear stress are discussed. The entrance length of the electroosmotic flow is longer than that of classical pressure-driven flow. The thickness of the electrical double layer (EDL) in the entry region is thinner than that in the fully developed region. The change of velocity profile is apparent in the entrance region, and the axial velocity profile is no longer flat across the channel height when the Reynolds number is large. © 2001 Academic Press

Key Words: electroosmotic flow; entrance length; Nernst–Planck equation; Navier–Stokes equation; CFD.

INTRODUCTION

Microchannel flows are often used in the separation of biological or chemical components, such as the separation of DNA in genetic engineering. In practice, the process is called electrophoresis (1, 2). It uses the principle of different mobility of charged particles to move them in opposite directions under an applied electric field. The induced flow motion is also called electroosmotic flow. When the flow enters the microchannel, there is a developing region called the entry region where the velocity distribution and skin friction show significant variations in the streamwise direction. There is a fully developed region where the velocity and skin friction show no variation in the streamwise direction. The electroosmotic entry flow physics has not been reported in the literature, and it is the purpose of this work to study the entry flow in a microchannel.

Several factors affect the microchannel flow physics. They are the electrical charge density, the electric field, and the zeta potential of the microchannel. Theoretically, the classical Navier–Stokes equations have to be modified to include those factors for the case of microchannel flow. Yang *et al.* (3) and Arulanandam and Li (4) used numerical methods to simulate fluid flow through a microchannel. Their physical models were based on (a) the Poisson–Boltzmann equation for the electrical double layer (EDL) potential, (b) the Laplace equation for the applied electrostatic field, and (c) the Navier–Stokes equations modified to include effects of the body force due to the interaction between electrical and zeta potential. Their numerical results of the model are in qualitative agreement with their experimental observations. They studied the microchannel flow in the fully developed region and neglected the entry region effects, and the charge density was assumed to be in the Boltzmann equilibrium distribution. Patankar and Hu (5) employed a similar model to investigate 3-D microchannel flow.

The electrophoresis separation usually requires injection of the sample into the microchannel. The entry region of the injection will influence the separation efficiency. To investigate the effect of the entry region on the flow characteristics and without assuming the Boltzmann equation for the charge density distribution, we performed calculations using the full Navier–Stokes equation and the Nernst–Planck equation. Our physical model is different from those in previous studies; namely, (a) the computational domain is from the entry region to the fully developed region, (b) the effects of the charge density on the fluid flow are considered without the assumption of Boltzmann distribution, and (c) we unify the applied electrical potential and zeta potential instead of using the superposition method. This physical model is believed to be better in describing the physical phenomenon of electroosmotic fluid flow in microchannel.

NUMERICAL METHOD

In this paper, our physical model is based on a combination of the Poisson equation for the applied electrical potential and the zeta potential of the fluid–solid boundary, the ionic concentration (n^+ , n^-) for the positive and negative ions of fluid, and the full Navier–Stokes equations modified to include effects of the electrokinetic force. By introducing the reference quantities, L_{ref} , U_{ref} , p_{ref} , and ρ_{ref} and defining $\bar{\rho}_e = \rho_e/n_0ze =$

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$(n^+ - n^-)/n_0$, $\bar{\psi} = ze\psi/k_bT$, $\bar{t} = tU_{\text{ref}}/L_{\text{ref}}$, $\bar{u} = u/U_{\text{ref}}$, $\bar{v} = v/U_{\text{ref}}$, $\bar{x} = x/L_{\text{ref}}$, $\bar{y} = y/L_{\text{ref}}$, $\bar{p} = (p - p_{\text{ref}})/(\rho U_{\text{ref}}^2)$, $\bar{\rho} = \rho/\rho_{\text{ref}}$, $\bar{n}^+ = n^+/n_0$, and $\bar{n}^- = n^-/n_0$, the dimensionless form of governing equations after dropping the head symbols can be written as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = -\frac{\kappa^2}{2} \rho_e \quad [1]$$

$$\begin{aligned} \frac{\partial n^\pm}{\partial t} + u \frac{\partial n^\pm}{\partial x} + v \frac{\partial n^\pm}{\partial y} \\ = \frac{1}{\text{Sc Re}} \left(\frac{\partial^2 n^\pm}{\partial x^2} + \frac{\partial^2 n^\pm}{\partial y^2} \right) \pm \frac{1}{\text{Sc Re}} \left[\frac{\partial}{\partial x} \left(n^\pm \frac{\partial \psi}{\partial x} \right) \right. \\ \left. + \frac{\partial}{\partial y} \left(n^\pm \frac{\partial \psi}{\partial y} \right) \right] \end{aligned} \quad [2]$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad [3]$$

$$\begin{aligned} \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \\ = -\frac{\partial p}{\partial x} + \frac{1}{\text{Re}} \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - Gx\rho_e \frac{\partial \psi}{\partial x} \end{aligned} \quad [4]$$

$$\begin{aligned} \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{\text{Re}} \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - Gx\rho_e \frac{\partial \psi}{\partial y}, \end{aligned} \quad [5]$$

where $\kappa = W \times K$, $K = (2n_0 z^2 e^2 / \varepsilon \varepsilon_0 k_b T)^{1/2}$ is the Debye-Huckel parameter, $1/K$ is the characteristic thickness of the EDL, $L_{\text{ref}} = W$, W is the channel height, $U_{\text{ref}} = \psi_{\text{inlet}} \varepsilon \varepsilon_0 \zeta / \mu L$, ψ_{inlet} is the activated potential at the inlet, and $\text{Re} = \rho_f U_{\text{ref}} L_{\text{ref}} / \mu = \rho_f (\frac{\psi_{\text{inlet}} \varepsilon \varepsilon_0 \zeta}{\mu L}) (\frac{L_{\text{ref}}}{\mu})$ is the Reynolds number. For the coordinate system of the microchannel, the x -coordinate is the fluid flow direction, the y -coordinate is perpendicular to the x -coordinate, and L is the length of the microchannel. Other variables and their definitions are listed in the Appendix. In most of the electrophoresis experiments, the microchannel is filled with aqueous electrolyte solution. The sample is injected into the channel, and then the electrical potential is activated to separate the sample. Consequently, the initial conditions are assumed to be that the velocity is static everywhere and the zeta potential of the wall is $\psi|_{\text{wall}} = \zeta$. The ionic concentration and potential are assumed to be in the Boltzmann equilibrium distribution initially. The boundary conditions are given as

$$\text{(In flow)} \quad \frac{\partial u}{\partial x} = 0 \quad \frac{\partial v}{\partial x} = 0 \quad \frac{\partial p}{\partial x} = 0 \quad \frac{\partial \psi}{\partial x} = C$$

$$n^+ = 1, \quad n^- = 1$$

$$\text{(Out flow)} \quad \frac{\partial u}{\partial x} = 0 \quad \frac{\partial v}{\partial x} = 0 \quad p(L, y) = \int_0^y Gx\rho_e \frac{\partial \psi}{\partial y} dy$$

$$\frac{\partial \psi}{\partial x} = C \quad \frac{\partial n^+}{\partial x} = 0, \quad \frac{\partial n^-}{\partial x} = 0$$

$$\begin{aligned} \text{(Walls)} \quad u = 0 \quad v = 0 \quad \frac{\partial p}{\partial y} = \frac{1}{\text{Re}} \frac{\partial^2 v}{\partial y^2} - Gx\rho_e \frac{\partial \psi}{\partial y} \\ \psi = \zeta + Cx \quad n^+ = e^{-\zeta}, \quad n^- = e^{\zeta}, \end{aligned}$$

where C is a constant. For the present problem we adopted $C = (0 - \psi_{\text{inlet}})/L$, where the outlet electric potential is grounded. The pressure boundary conditions along the y -direction are obtained from the y -momentum equation. They are not simply $\partial p / \partial y = 0$. Surface potential consists of zeta and applied electric potential. We assume that the surface potential is a simple superposition of the two potentials. Boundary conditions for n^+ and n^- at wall surfaces are assumed to be the values of nondimensional Boltzmann distributions. Discussion on other boundary conditions has been reported in Yang *et al.* (6). The governing Eqs., [1]–[5] must be solved simultaneously. We apply the artificial compressibility method (7, 8) by adding a time derivative pressure term to the continuity equation. To solve these equations, the finite difference method and alternative direction implicit (ADI) (7, 8) scheme are used.

RESULTS AND DISCUSSION

In the previous section, the general equations and boundary conditions were discussed. Here, we assume that the microchannel is made of silicon, the working fluid is water, and the electrolyte solution is KCl. The physical and electrical properties of liquid are the concentration of electrolyte $C = 10^{-5}$ – 10^{-6} M (9), the bulk concentration $n_0 = 6.022 \times 10^{20}$ – 6.022×10^{21} m $^{-3}$, the dielectric constant $\varepsilon = 80$, the zeta potential of the channel wall $\zeta = -75$ mV, the double-layer thickness parameter $\kappa = 3$ – 32 , the viscosity of the fluid $\mu = 10^{-3}$ N · s/m ^{2} , and the Schmidt number of the charge density $\text{Sc} = 10^5$. The ionic concentration and velocity fields are influenced by the electric field intensity. Therefore, it is obvious that the Reynolds number and velocity are functions of the electric field intensity. The entry length is determined by the interaction between the applied electrical potential and the net charge density. Hence, this discussion is directed to the influence of the charge density on velocity variation in the entry region.

The electroosmotic flow is fluid motion driven by an applied electrical potential, which quite differs from the traditional pressure-driven flow. In pressure-driven flow, when the velocity field is in a steady state, the velocity profile is parabolic in shape in the fully developed region. Many discussions have been recorded in classical fluid dynamics. In electrically driven flow, an applied electric field acts on the net fluid charge near the wall to produce a body force that drives fluid motion. Figure 1 shows typical plots of the velocity vector and net charge density contours at different time instants in the electroosmotic flow when $\text{Re} = 2$. Recall that the following initial conditions are used: the fluid fills the channel and the fluid is static everywhere. At this time, the zeta potential of the channel wall influences the fluid charge distribution to form an electrical double layer

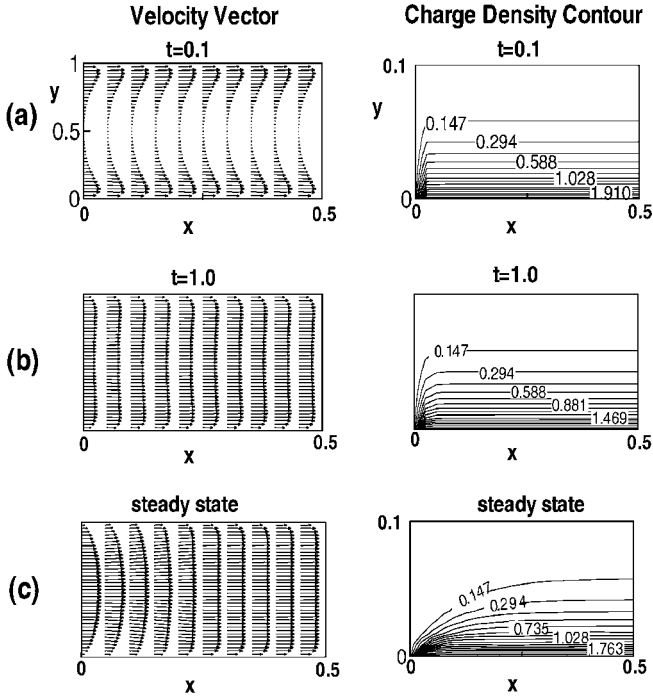


FIG. 1. The velocity vectors and charge concentration contour at time instant (a) $t = 0.1$, (b) $t = 1.0$, and (c) steady-state flow when $Re = 2$ and $\kappa = 32$.

(EDL), and the potential is a Boltzmann distribution. The fluid charge is therefore already attracted near the channel wall before the electrical field along the channel is activated. Notice that the interaction between the fluid charge of the near wall and the electric field is the main factor driving the fluid motion. There is no driving force near the center of the channel. The fluid motion therefore starts near the wall in the electroosmotic flow as shown in Fig. 1a. The velocity profile and net charge density field are gradually formed in a transient manner as shown in Fig. 1b. Since the fluid contains no charge far from the wall, as shown in the net charge density field, there is no driving force in this region. The fluid motion in this region is dragged by the viscous effect. Figure 1c shows the velocity and net charge density distributions at steady state. As the velocity field grows, the EDL at the entry region becomes thinner due to convection effects. The growth of the net charge density boundary layer along the streamwise direction is clearly seen. The boundary layer of the net charge density distribution influences the velocity distribution in the entry region. At the entry point $x = 0$, the velocity profile is similar to a parabolic shape. The fluid charge density is diminished by the convection effect at this location. Consequently, the cross-section is not influenced by the body force but is affected by the downstream fluid motion, which produces a suction effect at the inlet entrance. This effect is similar to a pressure-driven force and produces a parabolic velocity profile. Meanwhile, the charged boundary layer gradually grows along the downstream of the channel and the velocity profile changes from parabolic to flat stage by stage.

To distinguish differences between the electroosmotic and pressure-driven flows in the entry region, we plot contours of streamline u - and v -velocity components for both flows under steady state in Fig. 2. Both flows have identical mass flow rate and Reynolds number ($Re = 2$). The value of the v -velocity component is about one order of magnitude smaller than that of the u -velocity component in the entry region. Prior to the fully developed region, the v -velocity component appears in the entire cross section. However, the v -velocity components show opposite directions in the two flows. The v -velocity becomes zero in the fully developed region. In the case of electroosmotic flow, the streamlines bend toward the wall surface in the entry region and then keep parallel to each other in the fully developed region, while the streamlines bend toward the channel center for classical pressure-driven channel flows.

Figure 3 shows detailed streamwise velocity profiles at several x -locations and their comparison with respect to the analytical solution at $\kappa = 32$ obtained by Patankar and Hu (5). The analytical solution is based on linear superposition in the applied electrical potential and zeta potential. Note that, in our present study, the applied potential and zeta potential are unified in the flow field except the wall boundary conditions in which zeta potential and electrical potential are superimposed. In practice, the magnitude of the zeta potential is much smaller than that of the electrical potential. The present result demonstrated that results

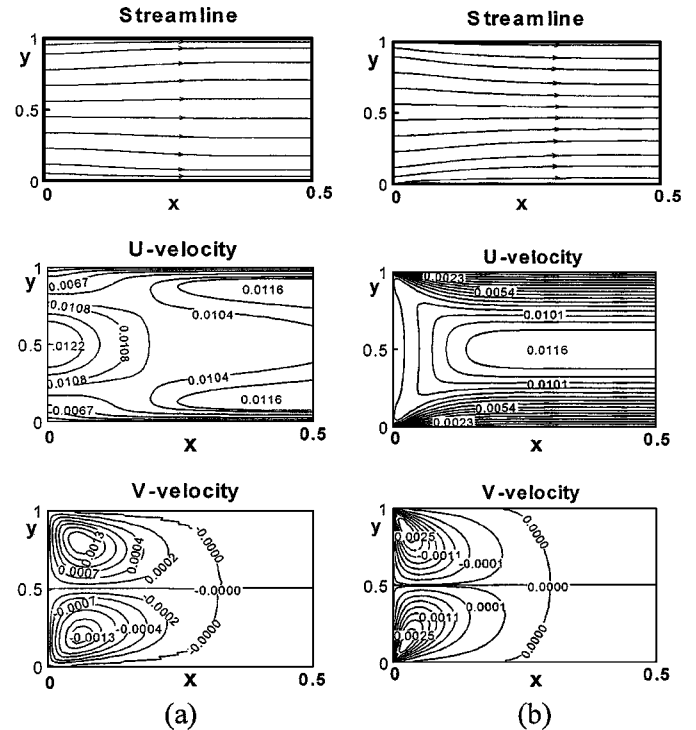


FIG. 2. Comparison of the streamline and u - and v -velocity contours with the same mass flow-rate in the entry region: (a) electroosmotic flow and (b) pressure-driven flow when $Re = 2$.

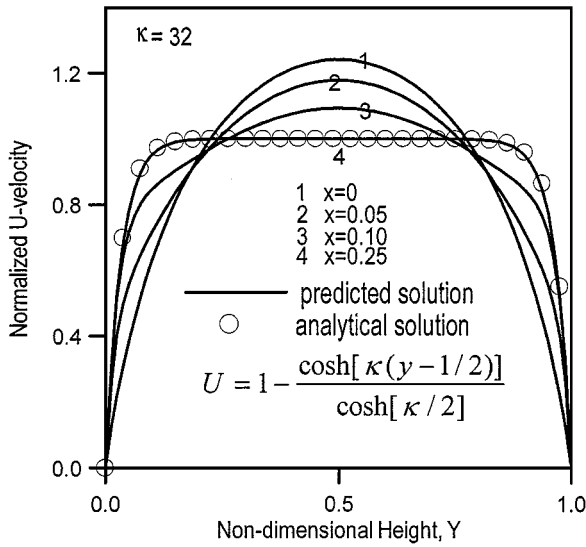


FIG. 3. Comparison between the computed result and analytical solution of the velocity distributions in the fully developed region when $\kappa = 32$.

obtained from both methods of superposition and unification of potential agree well with each other in the fully developed region. The figure also shows variations of velocity profiles in the entry region.

In the electrophoresis separation, the flow vorticity influences the efficiency of separation due to fluid mixing. The flatter the velocity profile is, the better the separation efficiency will be. It is very important to control the entry length and the velocity distribution in the cross section. The Reynolds number (electric potential) is a very significant parameter for controlling this factor. Figure 4 shows a plot of the velocity vector and the velocity profile in the cross section with the different Reynolds numbers in the fully developed region. In Fig. 4a, the velocity field attains the fully developed stage very soon when $Re = 0.5$. The entry region effect shows nearly no impact on the velocity distributions, and the computed velocity profile is flat in the fully developed region. However, there is a clear difference in velocity distributions in the entry region when $Re = 5$ and the velocity profiles have a concave shape in the fully developed region. Figure 4b reveals the velocity profile in the cross section in the fully developed region at Reynolds numbers between 0.5 and 5. From this figure, it is obvious that the velocity profiles are flat when $Re = 0.5$ and 2. The profiles have a concave shape when $Re = 3$ and 5. When the Reynolds number is small, its viscosity effect is larger and the viscous interaction between fluids is stronger. The fluid near both walls drags the fluid of the central channel, and the velocity profile becomes flatter. In contrast, when the Reynolds number is larger (i.e., corresponding to a larger electric field), the electrokinetic force drives the charges in the boundary layer to move faster than the fluid away from the boundary, which has less or no charges. Therefore, the velocity profile will show a concave shape.

Figure 5 presents a plot of the net charge density boundary layer for various Reynolds numbers. The thickness of the net charge density boundary layer is constant in the fully developed region because the EDL thickness and zeta potential of the wall are constant. The thickness is taken at the place where the value of the net charge density equals 1% of n_0 . This figure shows that the entry length of the charge density field is dependent on the Reynolds number. The fluid contains no charge before the entrance ($x = 0$) of the microchannel. But the liquid ions near the wall will form a net charge density boundary layer after entering the microchannel. The growth of the net charge density layer is similar to that of a classical velocity boundary layer, and the entry length is a function of the Reynolds number. From Fig. 5, it is clear that the entry length increases as

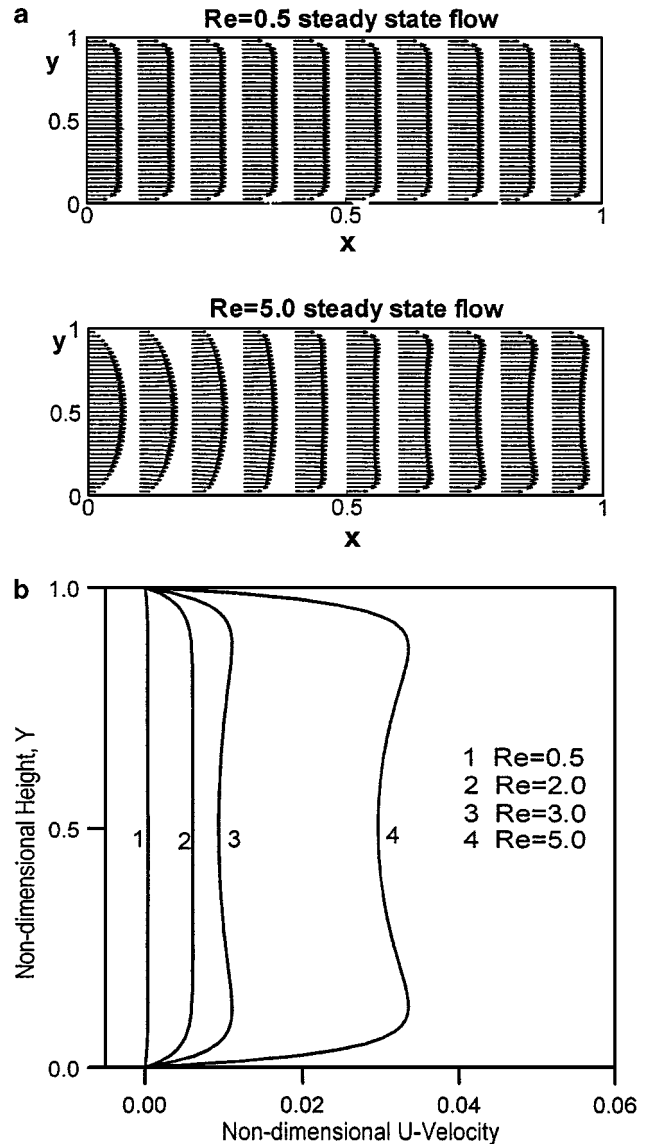


FIG. 4. (a) Comparison of the velocity vectors for $Re = 0.5$ and (b) Comparison of the velocity distributions for various Re values.

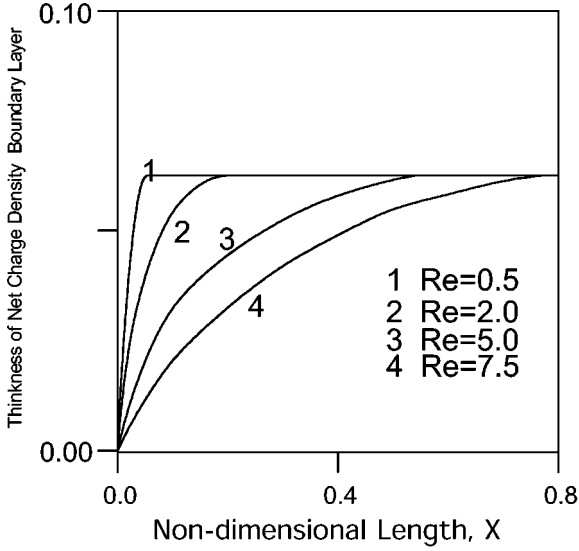


FIG. 5. Comparison of the thickness of net charge density boundary layer distributions for various Re values.

the Reynolds number increases. There are very important relations among the fluid viscosity, electrical potential, and zeta potential of the wall. Figure 6 shows the entry length of various Reynolds numbers for electrical potential and pressure-driven flows with the same mass flow rate. The entry length of a classical fluid dynamic flow is about $L \approx 0.06\text{Re}$. In an electroosmotic flow, the entry length is approximately $L \approx 0.11\text{Re}$. The difference is because, in addition to the viscous drag in pressure-driven flows, the attraction between the zeta potential of the wall and the charge of the liquid in electroosmotic flow must be overcome as well. Consequently, the entry length must be

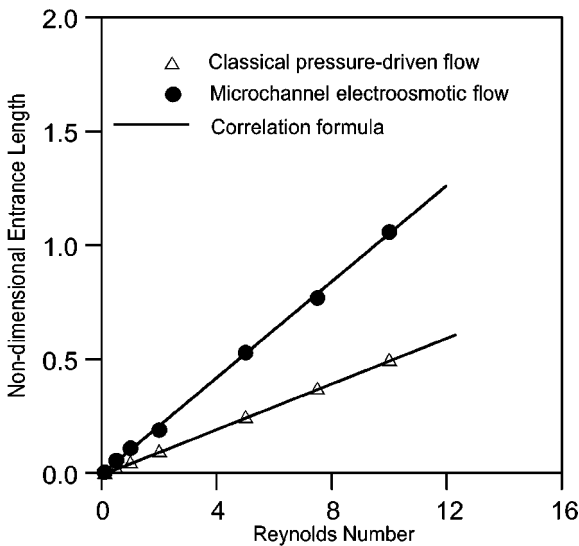


FIG. 6. Comparison of the entry length between the electroosmotic and pressure-driven flow with the same mass flow-rate.

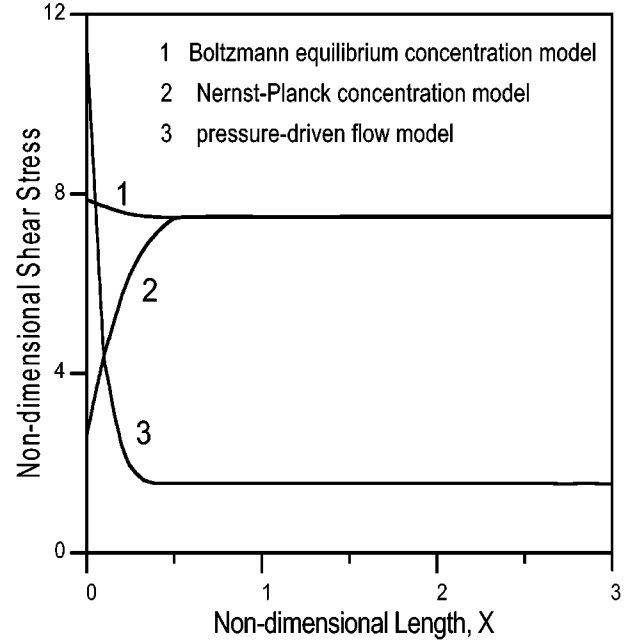


FIG. 7. Comparison of the wall shear stress between the electroosmotic and pressure-driven flows with the same mass flow-rate.

a longer distance to complete the fully developed phenomenon for electroosmotic flows. In Fig. 7, the wall shear stress as given by

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_{y=0}$$

was computed with the same mass flow rate for three cases, namely, (1) the electroosmotic flow with a Boltzmann equilibrium concentration model, (2) the electroosmotic flow with a Nernst–Planck concentration model, and (3) the pressure-driven flow model. For the case of pressure-driven flow, the flow starts with a uniform axial velocity profile and then progresses to a parabolic shape in the fully developed region. The shear stress changes dramatically in the entry region and then remains constant in the fully developed region. For the cases of electroosmotic flows, axial velocity profile distributions have significant variations near the wall resulting in higher shear stress than the pressure-driven flow in the fully developed region. Due to the charge density being assumed to be the Boltzmann distribution and the constant EDL thickness for the first case, the change of the velocity profile is not apparent in the entrance region. The entrance length is shorter for this case, compared to that of the second case. The shear stress has identical results for both the first and second cases in the fully developed region. But in the entrance region, the variation of the velocity profile is very conspicuous for the second case, as discussed in Fig. 1. Consequently, the shear stress changes from a small value in the entrance region to a large value in the fully developed region.

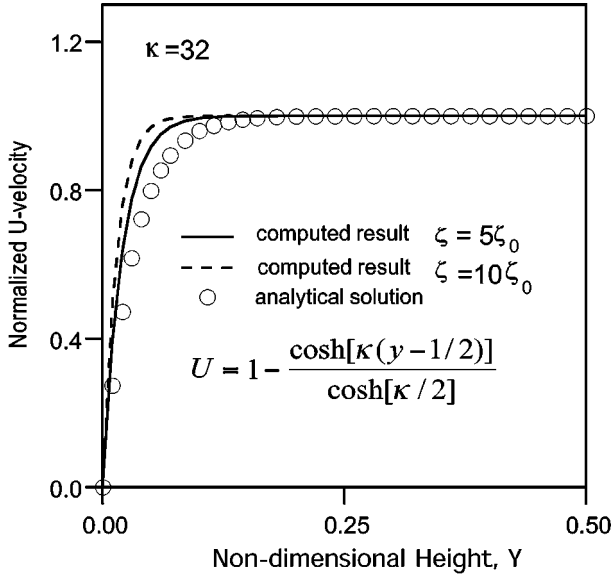


FIG. 8. Comparison between the computed result and classical superposition solution of the velocity distributions in the fully developed region when $\zeta = 5\zeta_0$ and $\zeta = 10\zeta_0$.

The present approach is novel in two aspects; namely, the applied electric potential and zeta potential are unified without using linear superposition and the Nernst-Planck equation is solved instead of the Boltzmann equation being used for charge density distribution. It is therefore interesting to investigate cases where the zeta potential can no longer be regarded as small

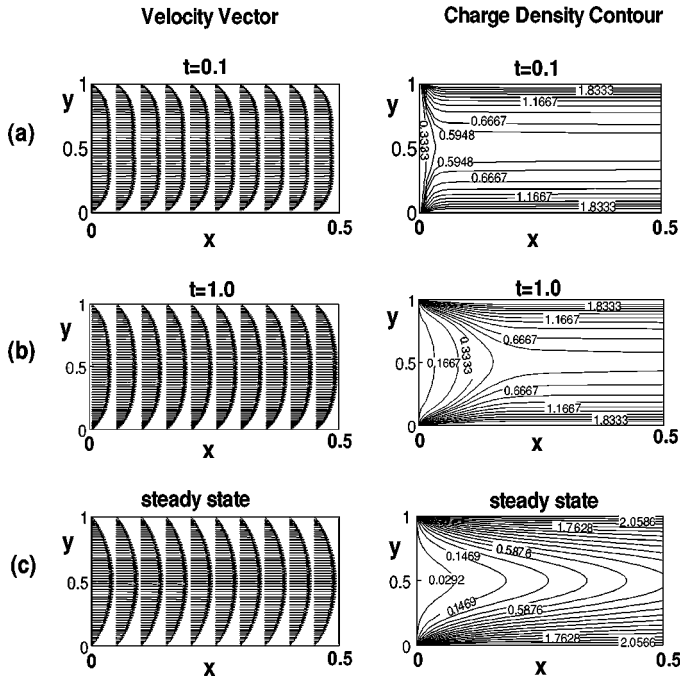


FIG. 9. The velocity vectors and charge concentration contour at time instant (a) $t = 0.1$, (b) $t = 1.0$, (c) steady-state flow when $Re = 2$ and $\kappa = 3$.

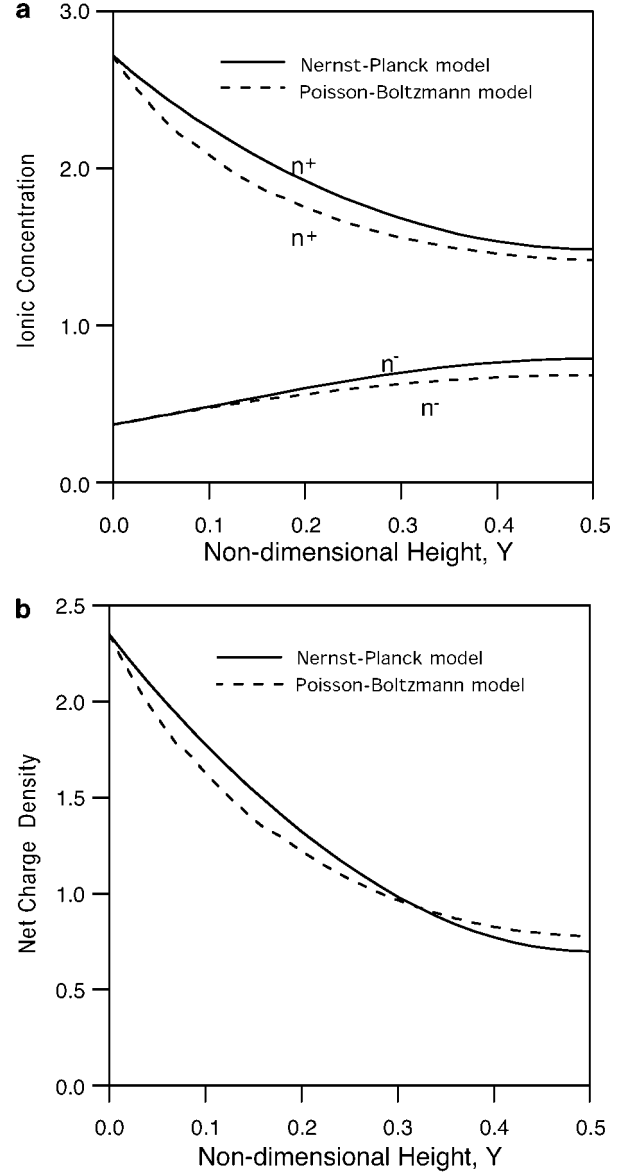


FIG. 10. Comparison of the concentration distributions between the Nernst-Planck model and the Poisson-Boltzmann model: (a) ionic concentration, (b) net charge density when $\kappa = 3$.

and the Boltzmann distribution is not valid when the EDL is overlapped in the channel. Figure 8 shows steady-state velocity comparisons between the present approach and the classical theory with several zeta potentials. We used $\zeta = -75 \text{ mV} = \zeta_0$ as a base, which gives a result nearly identical to that from the classical method as shown in Fig. 3. In Fig. 8, when $\zeta = 5\zeta_0$ and $\zeta = 10\zeta_0$, results for the u -velocity different from those of the classical superposition method appear near the walls. The small zeta-potential is therefore an inherent restriction on the classical superposition method. With the present unified approach, there is no such restriction on the magnitude of the zeta-potential.

As indicated by Qu and Li (10), when the EDL is overlapped in the channel, the Boltzmann equation for charge density may lead

to an inaccurate result. To demonstrate this issue, we performed a calculation for $\kappa = 3$ (corresponding to W equal to $3/32$ of the case of $\kappa = 32$). We assumed the surface zeta potential to be the same as in the case of $\kappa = 32$. In such a short separation distance of the parallel plates, the EDL is overlapped in the channel. Figure 9 shows velocity vectors and net charge density contours at several instants during the calculation. The net charge density is full of the channel, which produces electrokinetic force when the applied electrical potential is activated in the inlet. The driving force therefore appears in the entire cross-section, in contrast to the case that the EDL has no overlap in the channel. The steady-state velocity profile shows a parabolic shape similar to those of the classical pressure-driven flows where the driving force also appears in the entire cross-section of the channel. Figure 10 shows results for ionic distributions of n^+ , n^- , and net charge density $\Delta n = n^+ - n^-$ across the channel from the two different approaches. It clearly shows that the net charge density is full of the entire cross-section of the channel. Certain differences are observed in the predicted results. The results show a similar trend reported by Qu and Li (10). With the present physical model by solving Nernst–Planck equation, there is no restriction on the EDL being overlapped or not.

SUMMARY

The flow physics in the entry region of an electroosmotic flow is numerically simulated. The model we have presented is based on the combination of the Poisson, Nernst–Planck, and full Navier–Stokes equations to allow the calculation of flow entrance length and charge boundary layer distribution and to influence parameters such as the Reynolds number. The treatment on the applied electrical potential and zeta potential is unified in the Poisson equation without using superposition. The present physical model provides more general applications compared to the classical superposition method and restrictions on using the Boltzmann equation. The results showed greater entrance length of the electroosmotic flow than for classical pressure-driven flow. The thickness of the electrical double layer (EDL) as calculated via the Nernst–Planck equations grows from zero at the flow inlet to the Boltzmann equilibrium distribution in the fully developed region. The velocity profile is no longer flat when the Reynolds number is large. With the same mass flow rate condition, electroosmotic flow has higher skin friction than pressure-driven flow. The solutions may provide useful information for designing biochips in today's microelectromechanical systems (MEMS) technology.

ACKNOWLEDGMENT

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APPENDIX: NOMENCLATURE

D	Diffusion coefficient of ions (m^2/s)
e	Electron charge, (C)
G_x	The ratio of the EDL energy to the mechanical kinetic energy $= 2n_0k_bT\rho_fW^2/\mu \text{Re}^2$
K	Debye–Huckel parameter (m^{-1})
K_b	Boltzmann constant (J/K)
L	Channel length ($\bar{L} = L/L_{\text{ref}}$)
n^+, n^-	Local concentration of positive and negative ions (m^{-3}) ($\bar{n}^+ = n^+/n_0$, $\bar{n}^- = n^-/n_0$)
n_0	Average concentration of positive and negative ions (m^{-3})
p	Pressure (Pa)
Re	Reynolds number $= \rho(\frac{\psi_{\text{inlet}}\varepsilon\varepsilon_0\zeta}{\mu L})(\frac{W}{\mu})$
Sc	Schmidt number $= \mu/\rho_f D$
T	Absolute temperature (K)
U_{ref}	Reference velocity $= \frac{\psi_{\text{inlet}}\varepsilon\varepsilon_0\zeta}{\mu L}$
u	Velocity component along the x -direction ($\bar{u} = u/U_{\text{ref}}$)
v	Velocity component along the y -direction ($\bar{v} = v/U_{\text{ref}}$)
W	Channel height ($\bar{W} = W/L_{\text{ref}}$)
z	Valence of ions
ζ	Surface zeta potential (V) ($\bar{\zeta} = ze\zeta/k_bT$)
ψ	Electrical potential (V) ($\bar{\psi} = ze\psi/k_bT$)
ρ_e	Electric charge density $= (n^+ - n^-)ze$, (C/m^3), ($\bar{\rho}_e = (n^+ - n^-)/n_0$)
ρ_f	density of fluid
μ	viscosity (Ns/m^2)
ε	dielectric constant, dimensionless
ε_0	permittivity of vacuum (C/mV)

REFERENCES

1. Hu, L., Harrison, J. D., and Masliyah, J. H., *J. Colloid Interface Sci.* **215**, 300 (1999).
2. Culbertson, C., Ramsey, R., and Ramsey, J. M., *Anal. Chem.* **72**, 2285, (2000).
3. Yang, C., Li, D., and Masliyah, J. H., *Int. J. Heat Mass Transfer* **41**, 4229 (1998).
4. Arulanandam, S., and Li, D., *Colloids Surf. A Physicochem. Eng. Aspects* **161**, 89 (2000).
5. Patankar, N. A., and Hu, H. H., *Anal. Chem.* **70**, 1870 (1998).
6. Yang, R.-J., Fu, L.-M., and Lin, Y.-C., *J. Colloid Interface Sci.* **239**, 98 (2001).
7. Chorin, A. J., *J. Comput. Phys.* **2**, 12 (1967).
8. Yang, R.-J., Jeng, Y.-R., and Fu, L.-M., *Comput. Fluid Dyn. J.* **7**, 397 (1999).
9. Weast, R., Astle, M. J., and Beyer, W. H., "CRC Handbook of Chemistry and Physics." CRC Press, Boca Raton, FL, 1986.
10. Qu, W., and Li, D., *J. Colloid Interface Sci.* **224**, 397 (2000).